Reducing Carbon using Regulatory and Financial Market Tools

Franklin Allen[*]   Adelina Barbalau[†]   Federica Zeni[‡]

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Abstract

We study the conditions under which debt securities that make the cost of debt contingent on the issuer's carbon emissions, similar to sustainability-linked loans and bonds, can be equivalent to a carbon tax. We propose a model in which standard and environmentally-oriented entrepreneurs can adopt polluting and non-polluting technologies, with the latter being less profitable than the former. A carbon tax can correct the laissez-faire economy in which the polluting technology is adopted by standard entrepreneurs, but requires sufficient political support. Carbon-contingent securities provide an alternative price incentive for standard entrepreneurs to adopt the non-polluting technology, but require sufficient funds to fully substitute the regulatory tool. Absent political support for the tax, carbon-contingent securities can only improve welfare, but the same is not true when some support for a carbon tax exists. Understanding the conditions under which the regulatory and capital market tool are substitutes or complements within one economy is an important stepping stone in thinking about carbon pricing globally. It sheds light, for instance, on how developed economies can deploy finance to curb carbon emissions in developing economies where support for a carbon tax does not exist.

[*]Imperial College London
[†]University of Alberta
[‡]The World Bank

1   Introduction

There is widespread scientific consensus that Earth's climate has warmed significantly since the late 1800s and human activities, primarily greenhouse gas emissions, are the primary cause. Consequently, the issue of reducing and pricing emission has risen on the agenda of policymakers and has been the subject of numerous debates. As illustrated in Figure 1, there is considerable heterogeneity across countries with respect to whether or not carbon pricing regulation is implemented and the form that it takes, with some countries adopting a carbon tax, others a cap-and-trade system, and a few others having adopted both.[1] There are many reasons behind this fragmented regulation. At the international level, there are complex considerations around what would constitute an equitable climate transition that takes into account the fact that the countries most exposed to climate damages are the ones that have contributed the least to global emissions, and are also the ones least equipped with the resources to finance the climate transition.[2] At the domestic level, the policy design and implementation are critically affected by a series of political constraints which depend on electoral preferences and concern for the environment, expectations of energy costs, and policymakers' incentives.[3]

Figure 1. Carbon Pricing Regulation
The figure captures the current state of carbon pricing regulation worldwide as downloaded from the up-to-date carbon pricing dashboard developed by the World Bank Group. Source *https://carbonpricingdashboard.worldbank.org*, accessed November 2022.
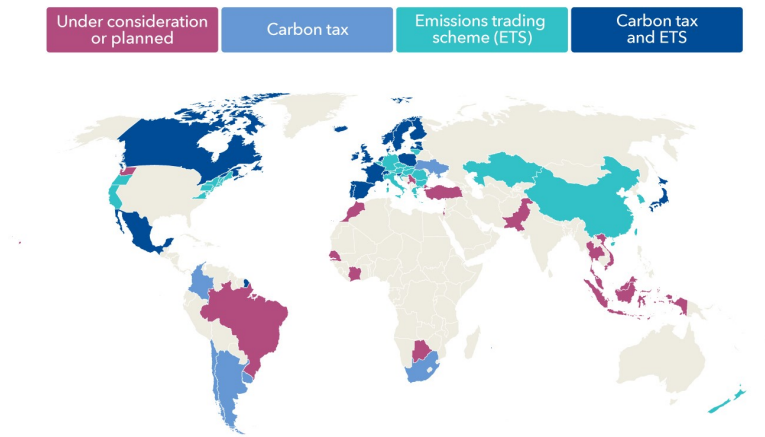
Source: WBG, IMF staff calculations, and national sources. Note: The boundaries and other information shown on any maps do not imply on the part of the IMF any judgment on the legal status of any territory or any endorsement or acceptance of such boundaries.

Even when regulation has been implemented, the carbon prices implied by the adopted regulatory tools are largely below the consensus level needed to incentivize the achievement of the Paris Agreement goal to remain below the 1.5°C degree rise in global temperature. Furthermore, the investment estimates needed

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[1]A carbon tax involves charging a tax on each unit of pollution. A cap-and-trade system involves capping the total quantity of emissions allowed, distributing rights to emitters within this total, and allowing them to trade the permits among themselves.

[2]A comprehensive discussion around these issues can be found in Nordhaus [2020].

[3]Prominent examples are the Washington State's two failed carbon tax referendums from 2016 and 2018, which are studied in detail in a recent work by Anderson, Marinescu, and Shor [2019].

to achieve this goal are significant and range from \$5 trillion per year by 2030 (World Resource Institute, 2021) to \$6.9 trillion per year (OECD, 2018). Many developing countries such as India, argue that developed countries that have been responsible for large emissions during their industrialisation over many years should be responsible for bearing most of the costs of the transition. Indeed, in 2009 developed countries committed to jointly mobilize \$100 billion a year by 2020 to help developing countries adapt to climate change, but these funds have been slow to come by and as of 2020 were still about \$17 billion short.[4]

The amount of financial resources that needs to be mobilized in order to support the climate transition is significant, and well beyond the scope of what governments can provide. Financial markets are now playing an increasingly important role by providing a platform through which investors and entrepreneurs can channel funds towards projects with environmental, social and sustainability-related outcomes. A prominent example is the market for sustainable debt securities, which has grown exponentially in recent years from a total issuance volume of \$109 billion pre-2012, to \$5,449 billion as of 2022Q3 (see Figures 2 and 3 below).[5] Of these, \$1,472 billion consist of sustainability-linked debt, a new class of instruments introduced only in 2018 which have an interest rate that is contingent on the issuer's performance against a sustainability-related target, which in most cases is represented by greenhouse gas emissions.[6]
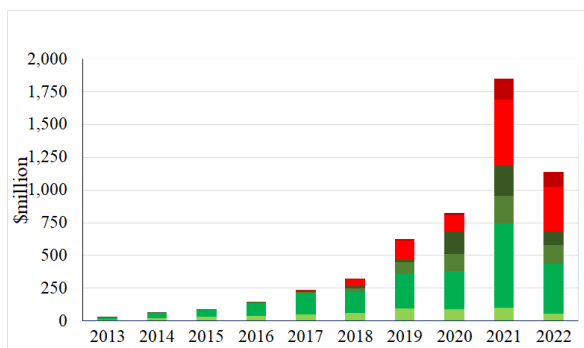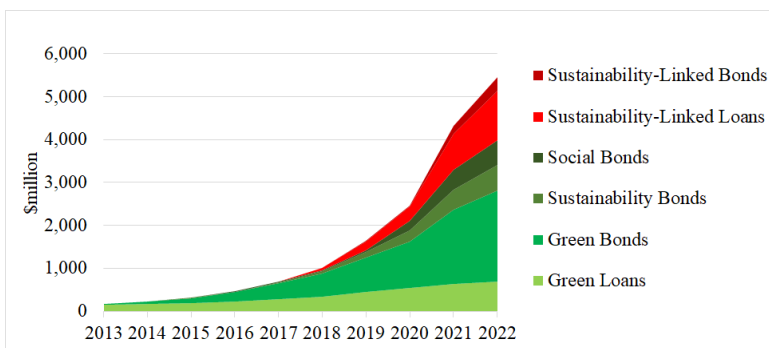

Figure 2. Sustainable Debt Issuance per Year

Figure 3. Cumulative Sustainable Debt Issuance

Importantly, the capital mobilized through sustainability-linked debt is orders of magnitude larger than the \$100 billion pledge to developing countries, and this form of carbon-contingent financing has a wider reach, being implemented in countries where support for regulation has been insufficient (see Figure 4 below). By combining the global nature of capital markets with the carbon-price incentives of regulation, these securities have the potential to be an important tool for reducing carbon.

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[4]Details can be found in the OECD report `https://www.oecd.org/climate-change/finance-usd-100-billion-goal/`

[5]This market comprises project-based securities such as green, social and sustainable bonds and loans, as well as outcome-based securities such as sustainability-linked loans and bonds which make the cost of debt contingent on outcomes such as the issuers' reduction in carbon emissions. Barbalau and Zeni [2022] provide a detailed analysis of the market and analyse the incentives for issuing these security classes.

[6]Details regarding the targets underlying sustainability-linked debt instruments can be found in, for instance, Barbalau and Zeni [2022] and Kölbel and Lambillon [2022].

Figure 4. Percentage of Sustainability-Linked Debt Issuance
This figure shows the geographical distribution of sustainability-linked debt (which includes corporate and government issued sustainability-linked loans and bonds) relative to all debt (corporate and government issued loans and bonds) issued since 2013. Data are collected from Bloomberg. A more intense shade of green indicates a higher proportion of sustainability-linked debt relative to total debt.
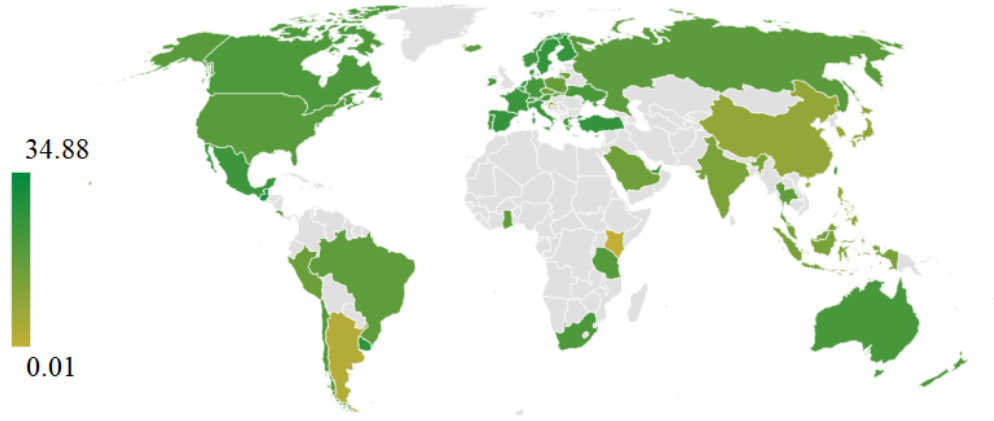
Motivated by this stylized evidence, in this paper we study the interaction between regulatory and financial market tools for pricing carbon within one economy, and focus in particular on the role of its population's wealth and environmental concerns. The regulatory tool we focus on is a carbon tax that can be implemented by the domestic regulator subject to a median voter political constraint that at least half of the voters are better off with the tax. The financial market tool is represented by carbon-contingent securities which have a payoff that increases (decreases) if the issuer's carbon emissions are in excess (deficit) of a predetermined target, in a manner that resembles the one observed in sustainability-linked debt instruments. The focus on a single economy is a necessary first step to study how regulation and financial markets jointly shape incentives to reduce emissions while abstracting from cross-country considerations such as international agreements and carbon leakage effects.

We start by proposing a baseline model which features standard and environmentally-oriented investors and entrepreneurs that are risk-neutral and behave atomistically. To simplify the exposition we refer to these as entrepreneurs throughout even though some of the time they are entrepreneurs, other times, they are investors in other entrepreneurs projects or both. Both standard and environmental entrepreneurs are exposed to climate shocks caused by global carbon emissions, but environmental entrepreneurs also internalize the negative impact of emissions associated with their actions. Each entrepreneur has endowments which she can either invest in polluting and non-polluting production technologies, with the latter being less profitable than the former, or lend to other entrepreneurs through carbon-contingent debt securities. There is a regulator that sets a carbon tax to maximize social welfare and who is subject to a median voter political constraint which means that it can only implement a tax if it is admissible for at least half

of the population.

The model predicts that in a laissez-faire economy without financial markets, standard entrepreneurs will invest in the polluting technology and environmental ones in the green, non-polluting technology. If exposure to climate shocks is higher than the profitability loss brought about by investing in the less profitable green technology, the regulator will find it optimal to implement a carbon tax and by doing so correct the laissez-faire economy, improve welfare and reduce emissions. However, the extent to which the tax can be enforced is subject to a political constraint that depends on the relative proportion and endowments of standard and environmental entrepreneurs.

If the carbon tax is not implemented, carbon-contingent financing from environmental to standard entrepreneurs arises. Carbon-contingent securities offer an alternative price incentive for standard entrepreneurs to switch to the green technology, but the extent to which these securities can fully substitute regulation depends on the funds of environmental entrepreneurs, who are effectively financing the transition. When the funds deployed are sufficiently large, the financial market solution can fully substitute regulation and is welfare enhancing independent of the stringency of the political constraint i.e. the support for the carbon tax. Relative to a carbon tax, the financial market solution achieves a higher welfare as it allows standard entrepreneurs to monetize green preferences, and environmental entrepreneurs to optimally increase their environmental impact. When the political constraint is binding and there is no support for a carbon tax, the financial market solution creates welfare gains even when the funds deployed by environmental entrepreneurs are small. However, the existence of financial markets for pricing carbon has the effect of decreasing support for regulation in equilibrium and can thus shift the economy from one that supports a carbon tax to one that does not. When that happens and the capital deployed through carbon-contingent financing is small and can only finance the transition of a small share of standard entrepreneurs towards the adoption of non-polluting technologies, there can be welfare losses.

The baseline model delivers useful insights, but cannot rationalize the empirical evidence that carbon-contingent financing co-exists with carbon pricing regulation (as suggested from Figures 1 and 4). Formally modelling the intensive-margin interaction between market-based and regulatory tools is necessary if one wants to derive an optimal carbon tax policy which accounts for the role of green finance in a realistic way. Therefore, we extend the model to allow for a continuum of entrepreneurs with heterogeneous environmental preferences, as well as a continuum of production technologies with a carbon intensity that can be reduced by incurring a convex carbon abatement costs.

In the continuous model, the regulator can implement a revenue-neutral tax which involves redistributing the revenues from the tax equally across all entrepreneurs. We first show that the issuance of carbon-

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contingent securities, the market-implied price of carbon, and the resulting emission abatement generated by financial markets are a decreasing function of the tax, suggesting again that the two tools can be used as substitutes. We then show that, in line with the linear model, the presence of financial markets make the carbon tax less appealing for the median voter, thereby reducing the probability of implementation of a given tax. We thus solve for the optimal revenue-neutral carbon tax in the combined presence of a financial market for pricing carbon and the median voter political constraint.

We find that, in the absence of political constraints, the tax is optimally lower than the Pigouvian benchmark to account for the amount of emissions reduction generated by financial markets in equilibrium, and welfare is strictly higher than the one achieved in an economy without financial markets. When political constraints are strongly binding, financial markets offer a welfare-improving alternative to the regulatory tool, although their ability to achieve the emission reductions generated by the Pigouvian benchmark depends on the lending capacity of entrepreneurs with stronger environmental concerns. However, when political constraints are weakly binding, the introduction of financial markets can lower welfare. This happens when the loss in political support for regulation is not offset by the value generated from the issuance of carbon-contingent securities. Furthermore, the potential welfare loss depends on the ex-post redistribution rule of the tax revenues, which plays an important role in determining the sensitivity of the median voter's preference for a given tax, suggesting that there is scope for a welfare-improving design of the ex-post compensations such as tax rebates.

The extended model is not only able to generate the observed co-existence of a carbon tax policy and carbon contingent finance, but it also rationalizes why countries with environmentally-oriented voters have both high carbon taxes and active sustainable finance markets. Importantly, our model predicts that in those highly regulated countries the share of emissions reduction achieved by financial markets is low relative to that achieved by the regulation, suggesting that carbon-contingent funds are best directed to markets without carbon taxes, where there is more abatement potential.

The rest of the paper is organized as follows: in Section 2 we provide a brief review of the related literature, underlying the original contribution of the work; in Section 3 we present and solve the baseline model; in Section 4 we present and solve the extended model; in Section 5 we conclude and discuss future directions of research in our framework.

2   Literature

This paper contributes to understanding how security design can enable financial markets to effectively complement government regulation in addressing the sustainability issues faced by the world. Our paper

can be broadly framed at the intersection between finance and environmental/climate economics.

The literature studying the interaction between financial markets and corporate behaviour has largely focused on understanding the conditions under which and channels through which investments by agents with pro-social and/or pro-environmental preferences can have an impact by reforming the firms. The channel most studied is the cost of capital channel. Notable papers in this literature stream include Heinkel, Kraus, and Zechner [2001] who study how exclusionary ethical investing impacts corporate behavior, Pastor, Stambaugh, and Taylor [2020] who study how shifts in customers' tastes for green products and investors' tastes for green holdings produce positive social impact, Oehmke and Opp [2022a] who study the conditions for impact in a context in which investors can relax firms' financial constraints for responsible production, and Landier and Lovo [2020] who study how ESG funds should invest to maximize social welfare in a setup in which financing markets are subject to a search friction. Whether investments can achieve a first-best outcome is also studied by Hong, Wang, and Yang [2021], who consider the role of investment mandates which involve restricting a fixed fraction of the representative investor's portfolio to hold firms that meet sustainability guidelines. Chowdhry, Davies, and Waters [2019] discuss security design related issues in the context of studying the conditions under which impact investments improve social outcomes when firms that cannot commit to social goals are jointly financed by profit and socially-motivated investors.

Dyck, Lins, Roth, and Wagner [2019] provide empirical evidence on the role of institutional investors in driving corporate environmental and social performance. Gupta, Kopytov, and Starmans [2022] highlight that socially responsible investors who value acquiring firms with high negative production externalities that they can reform, create trading gains that can actually cause a potential delay in reform. Broccardo, Hart, and Zingales [2022] emphasize a governance rather than a cost of capital channel, in a setup in which investors' preferences are alike those of a social planner internalizing global externalities. In most of these papers investors are big or they act as if they are big, whereas we examine atomistic investors that take global emissions as given. We are also effectively studying a cost of capital channel and abstract from corporate governance and a firm's decision to reform by taking the technologies as given and only looking at which will be financed in equilibrium. Our focus is instead on the role of regulation, which is absent in all the works cited above.

The literature stream that our paper is most related to is the one at the intersection of finance and corporate behavior, but which also brings regulation into the picture. Heider and Inderst [2021] examine the optimality of a uniform cap-and-trade policy when firms need costly external financing, and there is heterogeneity across firms and sectors. Biais and Landier [2022] study the complementarity between firms, which can invest in green technologies, and government, which can impose emission caps but has

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limited commitment power. They find a role for a large fund that can tilt the equilibrium towards caps by engaging with firms to foster investment in green technologies. Ramadorai and Zeni [2021] find that firms' abatement actions depend greatly on their beliefs about climate regulation, and that both informational frictions and reputational concerns can amplify responses to climate regulation, increasing its effectiveness. Huang and Kopytov [2022] show that in the presence of socially responsible investors, pollution can increase with regulation stringency because regulation reshapes firms' shareholders composition and makes polluting firms' shareholders less averse to holding polluting shares. Oehmke and Opp [2022b] study the role of green capital requirements for banks and show that capital regulation is a less effective tool to address carbon externalities that manifest themselves outside of the banking sector.

Financial frictions or actors are present, but all these papers are predicated on an implicit complementarity between finance and regulation, in that they both exist and the question is how they interact. We are, to the best of out knowledge, the first ones to note that a specific security design can substitute regulation. In doing so we build on the work of Barbalau and Zeni [2022] who study the trade-offs related to designing green debt securities as project-based contracts that specify ex-ante the projects that the proceeds will be allocated to, and outcome-based contracts that do not impose constraints on the use of proceeds but embed contingencies that ensure commitment to outcomes. We show that a carbon-contingent security design can fully substitute a carbon tax for which there is insufficient political support if the capital deployed through such instruments is sufficiently high.

More broadly, our paper relates to the large literature in climate economics that tackles the issue of pricing carbon, by emphasizing the value of using prices to reduce carbon emissions.[7] Stavins [2020] provides a very good overview of price (tax) and quantity (cap-and-trade) instruments for pricing carbon, discussing the dimensions along which these instruments differ and the features that make them equivalent. Goulder and Schein [2013] make a distinction between endogenous carbon pricing tools such as "pure" cap-and-trade systems that imply a market-based volatile carbon price, and exogenous pricing tools such as a carbon tax and a "hybrid" option (a cap-and-trade system with a price ceiling and/or price floor). They discuss the relationship between these tools, exploring the dimensions along which they are equivalent as well as when they have different impacts. Our contribution is to bring the financial sector into the conventional carbon picing analysis.

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[7]This is based on integrated assessment models which describe the global interplay between the economy and the climate, and are aimed at calculating the social cost of carbon, as well as quantifying mitigation scenarios for policy-making.

3 Simple Model

We start with a simple linear model featuring two technologies, two time periods $t = 0, 1$, two types of entrepreneurs[8] (standard and environmentally-oriented), and a regulator which sets a carbon tax to maximize social welfare given the total sum of utilities.

The two technologies take as input capital I to produce output y and carbon emissions e. They differ as follows:

(i) the polluting technology, indexed by π, yields output y_π and emissions e_π given by

$$y_\pi = \pi I \qquad \text{and} \qquad e_\pi = I,$$

where $\pi > 1$ is a production parameter;

(ii) the non-polluting or green technology, indexed by g, yields output y_g and zero emissions

$$y_g = gI \qquad \text{and} \qquad e_g = 0,$$

with g a green production parameter which satisfies $1 < g < \pi$.

There are two types of risk-neutral entrepreneurs indexed by $i = 1, 2$, namely:

(i) standard entrepreneurs, indexed by $i = 1$, who form a proportion θ of the population, are each endowed with capital h_1, and have utility

$$U_1 = C_1 - \lambda E,$$

where λ is a climate exposure parameter which captures the impact of the total emissions in the economy, $E = \theta e_1 + (1 - \theta)e_2$, which are a weighted sum of the emission produced by standard and environmental entrepreneurs, e_1 and e_2 respectively;

(ii) environmentally-oriented or green entrepreneurs, indexed by $i = 2$, who form a proportion $1 - \theta$ of the population, have capital h_2 and utility

$$U_2 = C_2 - \lambda E - \eta e_2,$$

where e_2 are emissions associated with their actions, η is a green preference parameter which is assumed to satisfy $\eta > \pi - g$. Note that whereas the environmental entrepreneurs dislike the emissions

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[8]As explained in the introduction, we refer to all these agents as entrepreneurs for simplicity of exposition. Initially, they are entrepreneurs but when we introduce financial markets, they can be investors or both.

associated with their actions and which they feel responsible for[9], both types of entrepreneurs are affected by total carbon emissions. The latter can be conceptualized as capturing a global climate shock that affects them irrespective of their preferences and over which they have no control. Thus, entrepreneurs are atomistic with respect to the global climate shock, which can be thought as a natural disaster or the negative effects of pollution on health which affect the entire population.

There is a regulator which maximizes utilitarian social welfare given by the sum of total utilities

$$W = \theta C_1 + (1 - \theta)(C_2 - \eta e_2) - \lambda E. \tag{1}$$

3.1 Laissez-Faire Benchmark

In the decentralized economy, entrepreneurs choose to produce output using the polluting or non-polluting technologies. Denote capital investment in the polluting and non-polluting technology by I_π and I_g, respectively, and denote agent i's green preference using η_i, which for the standard entrepreneur $i = 1$ takes the value $\eta_1 = 0$ and for the environmental entrepreneur $i = 2$ takes the value $\eta_2 = \eta$. Recall that emissions are only produced by the investment in the polluting technology, that is $e_\pi = I_\pi$ and $e_g = 0$. Thus, entrepreneur i's problem of allocating its endowment to the polluting and the green technology, is

$$U_i^* = \max_{I_\pi, I_g} \pi I_\pi + g I_g - \eta_i I_\pi - \lambda E \text{ such that } I_\pi + I_g \leq h_i. \tag{2}$$

Given we assumed $\pi > g > 1$, and $\eta > \pi - g$, it is immediate to see that the standard entrepreneur $i = 1$ will invest all available capital in the polluting technology, $I_\pi^* = h_1$, whereas the environmental entrepreneur will invest all capital in the non-polluting technology, $I_g^* = h_2$.

Taking account of such choices, the utility of the standard entrepreneur is

$$U_1^* = \pi h_1 - \lambda E^*$$

while the utility of the green entrepreneur is

$$U_2^* = g h_2 - \lambda E^*.$$

Aggregate emissions are

$$E^* = \theta e_1^* + (1 - \theta) e_2^* = \theta h_1,$$

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[9]This assumption is in line with work by Hart and Zingales [2017] that assumes that individuals put some weight on doing the right or socially efficient thing if they feel responsible for the action in question.

and the utilitarian social welfare is

$$W^* = \theta U_1^* + (1 - \theta)U_2^* = \theta\pi h_1 + (1 - \theta)gh_2 - \lambda\theta h_1.$$

3.2 Carbon Tax

Suppose that the regulator can alter the laissez-faire economy by imposing a tax τ on the emissions produced by the polluting technology π, and by doing so alter the investment decisions of the entrepreneurs. The utilitarian social welfare as a function of the tax is

$$W^\tau = \theta U_1^\tau + (1 - \theta)U_2^\tau + R^\tau \tag{3}$$

with U_1^τ and U_2^τ the utilities of the standard and environmental entrepreneurs, respectively, evaluated at their optimal investment choices given the tax τ, and $R^\tau = \tau(\theta e_1^\tau + (1 - \theta)e_2^\tau)$ the revenues from the tax, with e_1^τ and e_2^τ the optimal emissions of the standard and environmental entrepreneurs given the tax.

It is straightforward to show that any tax $\tau \geq 0$ will not change the actions of the environmental entrepreneur relative to the benchmark laissez-faire economy in which the green technology is adopted. It is therefore sufficient to focus on the standard entrepreneur's problem, which in the presence of the tax becomes

$$U_1^\tau = \max_{I_\pi, I_g} gI_g + (\pi - \tau)I_\pi - \lambda E^\tau \quad \text{such that} \quad I_\pi + I_g = h_1. \tag{4}$$

Optimal investment choices given the tax τ are

$$
\begin{aligned}
I_g^\tau &= h_1 \quad &\text{and} \quad I_\pi^\tau = 0 \quad &\text{if} \quad \tau \geq \pi - g \\
I_g^\tau &= 0 \quad &\text{and} \quad I_\pi^\tau = h_1 \quad &\text{otherwise,}
\end{aligned}
\tag{5}
$$

and the emissions associated with the standard entrepreneur's choices are $e_1^\tau = 0$ if $\tau \geq \pi - g$, and $e_1^\tau = h_1$ otherwise. Substituting the utilities U_1^τ and U_2^τ into the utilitarian social welfare in (3) and re-arranging, we have

$$
W = \begin{cases}
W^\tau = \theta gh_1 + (1 - \theta)gh_2 & \text{if} \quad \tau \geq \pi - g \\
W^* = \theta\pi h_1 + (1 - \theta)gh_2 - \lambda\theta h_1 & \text{otherwise.}
\end{cases}
\tag{6}
$$

Thus, implementing a carbon tax that is sufficiently high, i.e. $\tau \geq \pi - g$, to incentivize the transition to the green technology yields a higher welfare if $\lambda > \pi - g$. Therefore, the optimal tax is $\tau = \pi - g$ if $\lambda > \pi - g$, and $\tau = 0$ otherwise.

We focus henceforth on the case in which $\lambda > \pi - g$, such that the tax is implemented. In this case, aggregate emissions are zero, $E^\tau = 0 < E^*$, and utilitarian social welfare is higher relative to the laissez-

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faire economy

$$W^\tau = \theta g h_1 + (1 - \theta) g h_2 > W^*. \tag{7}$$

The utility of the green entrepreneur is higher than in the laissez-faire economy

$$U_2^\tau = g h_2 > U_2^* = g h_2 - \lambda \theta h_1 \tag{8}$$

so this class of entrepreneurs always supports the tax. On the other hand, the utility of the standard entrepreneur is higher than in the laissez-faire economy

$$U_1^\tau = g h_1 > U_1^* = (\pi - \lambda \theta) h_1 \tag{9}$$

provided that $\lambda \theta > \pi - g$. In the presence of the tax the aggregate climate shock no longer affects entrepreneurs' utilities since there are no emissions (creating a utility gain proportional to the fraction of standard entrepreneurs θ), but the tax shifts their investment choice to the less profitable green technology (creating a utility loss proportional to $\pi - g$). Whether standard entrepreneurs support the tax depends on this trade-off.

Therefore, the environmental entrepreneurs will always support the tax but the standard entrepreneurs will only support the tax if $\lambda \theta > \pi - g = \tau$.

Political Constraint. As discussed in the introduction, an important issue is the requirement that the regulation has political support. The regulator is subject to a political constraint in the sense that it can only implement a tax that makes at least half of the population better off. So there is sufficient political support for the tax if the median voter is an environmental type, i.e. $\theta < 0.5$, or if the median voter is a standard entrepreneur type, i.e. $\theta > 0.5$ but the tax satisfies $\tau < \lambda \theta$. Any tax larger than $\lambda \theta$ will not garner sufficient political support so this can be thought of as a threshold.

Support for regulation exists if the optimal tax chosen by the regulator is lower than the median voter tax threshold, defined as

$$\bar{\tau} = \begin{cases} \pi - g & \text{if } \theta < 0.5 \\ \pi - g & \text{if } \theta > 0.5 \quad \text{and} \quad \pi - g < \lambda \theta \\ 0 & \text{otherwise.} \end{cases} \tag{10}$$

Formally, the regulator must solve a constrained maximization problem of the type

$$\max_\tau \ W^\tau \ \text{such that } \tau \leq \bar{\tau} \tag{11}$$

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which states that the optimal tax should be at most equal to that supported by the median voter.

PROPOSITION 1. *Suppose that $\lambda > \pi - g$ such that the implementation of the carbon tax is desirable. Then if the median voter is an environmentally-oriented type $\theta < 0.5$, then the tax $\tau^o = \pi - g$ achieves the unconstrained optimum in* (11) *and social welfare is higher relative to the laissez-faire*

$$W^{\tau^o} = \theta g h_1 + (1 - \theta) g h_2 > W^*.$$

If the median voter is a standard type $\theta > 0.5$, then either $\lambda\theta > \pi - g$, in which case $\tau^o = \pi - g$, or $\tau^o = 0$, in which case

$$W^{\tau^o} = \theta \pi h_1 + (1 - \theta) g h_2 - \lambda \theta h_1 = W^*.$$

We have thus derived the optimal carbon tax policy in the absence of financial markets. In the next subsection we will consider the regulator's problem when financial markets for pricing carbon and agents can lend or borrow through carbon-contingent debt securities.

3.3 Carbon-Contingent Financing

So far, we have studied each entrepreneur's decisions assuming access to own capital only, represented by their endowments $h_i, i = 1, 2$. In what follows, we allow for external financing and essentially consider capital structure choices. Specifically, we introduce carbon-contingent debt securities similar to those observed in the market for sustainable finance and allow entrepreneurs to borrow and lend by issuing and purchasing these securities, respectively. In this setup, we assume that entrepreneur i can issue, at time $t = 0$, a debt security with principal value d_i and payoff at time $t = 1$ given by

$$\bar{r} d_i - \rho(\bar{e}_i - e_i), \tag{12}$$

where $\bar{r}$ is a fixed interest rate, e_i denotes entrepreneur i's emissions at time $t = 1$, and $\bar{e}_i$ the benchmark emissions set at time $t = 0$. These benchmark emissions $\bar{e}_i$ are essentially the counterfactual of what emissions would be in the absence of external financing. This return specification is analogous to that underlying sustainability-linked loans and bonds, which feature a fixed interest rate component and a variable component that is contingent on the deviation of realized emissions from a benchmark that is agreed at contract issuance. If realized emissions are higher than the benchmark, i.e. $e_i > \bar{e}_i$, then the interest rate in (12) increases and viceversa.

We first outline the issuer's problem and the lender's problem. Then, we derive the equilibrium fixed rate $\bar{r}$ and the contingent rate ρ as a function of entrepreneurs' preferences and endowments.

The Issuer's Problem. Consider first the case of the environmentally-oriented entrepreneur $i = 2$, whose benchmark emissions are $\bar{e}_2 = 0$. Upon issuance of the debt security, he will face the following investment problem

$$U_2 = \max_{I_\pi, I_g} \pi I_\pi + g I_g - (\eta + \tau) I_\pi - \bar{r} d_2 - \rho I_\pi - \lambda E \text{ such that } I_g + I_\pi \leq h_2 + d_2. \tag{13}$$

Since $\eta > \pi - g$, the environmental entrepreneur will continue to invest only in the green technology (i.e. $I_g = h_2 + d_2$) for any tax $\tau \geq 0$ or contingent rate $\rho \geq 0$, and so there will be no contingent component associated with the payoff in (12), which will simply degenerate into a fixed payoff $\bar{r} d_2$. In this economy, the supply of capital is provided by the standard entrepreneurs, the interest rate $\bar{r}$ is set such that these standard investors are just indifferent between lending to the environmental entrepreneurs or investing in their preferred technology. Therefore, we have that $\bar{r} = \pi$ if $\tau = 0$, and $\bar{r} = g$ if $\tau = \pi - g$. Hence, for the environmental entrepreneur $i = 2$, it is never strictly optimal to borrow external funds from the standard entrepreneur $i = 1$ because the interest rate to be repaid is at least as much as the return on their preferred investment, i.e. $\bar{r} \geq g$. We assume henceforth that when indifferent on the extensive margin, that is, when indifferent about raising or not external finance, the environmental entrepreneur always prefers to use internal finance only.

Consider now the case of the standard entrepreneur $i = 1$. If there is a carbon tax $\tau = \pi - g$, then the entrepreneur's benchmark emissions are $\bar{e}_1 = 0$, and the problem is similar to that of the environmental investor $i = 2$, and it is never strictly optimal for the standard entrepreneur $i = 1$ to raise external financing. On the other hand, if there is no carbon tax $\tau = 0$, then benchmark emissions are $\bar{e}_1 = h_1$ and the standard entrepreneur can profit if she reduces emissions $e_1 < \bar{e}_1$. The problem solved by the standard entrepreneur if she were to borrow d_1 through the issuance of carbon-contingent debt

$$U_1 = \max_{I_\pi, I_g} \pi I_\pi + g I_g - \bar{r} d_1 + \rho(h_1 - I_\pi) - \lambda E \text{ such that } I_g + I_\pi \leq h_1 + d_1, \tag{14}$$

which yields solution $I_g = h_1 + d_1$ if $\rho \geq \pi - g$, and $I_g = 0$ otherwise.[10] If the price of carbon implied by the carbon-contingent debt contract is sufficiently high to incentivize the transition to the green technology, i.e. $\rho \geq \pi - g$, the standard entrepreneur's utility is

$$U_1 = g(h_1 + d_1) - \bar{r} d_1 + \rho h_1 - \lambda E \geq \pi h_1 + g d_1 - \bar{r} d_1 - \lambda E, \tag{15}$$

whereas in the case where the contingent rate is not sufficiently high to incentivize switching to the green

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[10]Here we implicitly assume that when indifferent on the intensive margin, the entrepreneur always prefers to implement the green technology. Relaxing the assumption does not change the equilibrium outcome.

technology, i.e. $\rho < \pi - g$, the standard entrepreneurs' utility upon borrowing is

$$U_1 = \pi(h_1 + d_1) - \bar{r}d_1 - \rho d_1 - \lambda E < \pi h_1 + g d_1 - \bar{r}d_1 - \lambda E. \qquad (16)$$

Thus, borrowing through a carbon-contingent security is optimal only for financing investment in the green technology. If the contingent rate is sufficiently high to make the adoption of the green technology optimal i.e. $\rho \geq \pi - g$ such that $I_g \neq 0$, then the entrepreneur is better off borrowing. Otherwise, the entrepreneur is strictly worse off borrowing at a less favorable rate and investing in the polluting technology.

We now determine the equilibrium market price of carbon implied by the lending rate ρ, and the supply of credit to the standard entrepreneur $i = 1$ by solving the lender's problem.

The Lender's Problem. Environmental entrepreneurs $i = 2$ decide the optimal amount of lending d_2, and invest the remainder $h_2 - d_2$ in the green technology. Lending via the carbon-contingent security entails providing capital d_2 at time zero, and receiving at time $t = 1$ a fixed return component, $\bar{r}d_2$, and a variable return component that is contingent on the carbon emissions that are brought about by the entrepreneur's actions, $\rho(\bar{e}_2 - e_2)$. This latter component is a function of the emissions generated by the borrowers, as will be detailed below, and captures the fact that the environmental entrepreneur internalizes the possibility that the capital lent will be invested in emissions generating activities. The environmental investor's problem when acting as a lender is

$$U_2 = \max_{d_2 \leq h_2} g(h_2 - d_2) + \bar{r}d_2 - \rho(\bar{e}_2 - e_2) + \eta(\bar{e}_2 - e_2) - \lambda E, \qquad (17)$$

where the first term is the return from investing in the green technology, the next two terms are the cash flows associated with the contingent security, and the subsequent term captures green preferences reflecting the emissions associated with the agent's actions. Note that if the realized emissions are lower than the benchmark, i.e. $e_2 < \bar{e}_2$, the utility of the entrepreneur decreases via the financial channel i.e. the variable part of the contingent-security payoff, but it increases via the green preference channel. In the case considered here, the standard entrepreneurs are the borrowers so benchmark emissions are the counterfactual emissions that would be generated by the class of standard investors absent borrowing and the carbon tax, $\bar{e}_2 = h_1$. Recalling that funding is made available by a proportion $1 - \theta$ of environmental entrepreneurs, while carbon emission reductions are generated by a proportion θ of standard entrepreneurs, the emissions internalized by the environmental entrepreneur are $(\bar{e}_2 - e_2) = \frac{\theta}{1-\theta}(\bar{e}_1 - e_1) = \frac{\theta}{1-\theta}(h_1 - e_1)$. The investor maximizes utility (17) subject to the financing constraint that the total financial returns

from the investments are non-negative

$$g(h_2 - d_2) + \bar{r}d_2 - \rho(\bar{e}_2 - e_2) \geq 0, \tag{18}$$

so while this class of investors is willing to reward emission reductions they will only do so up to the point that they deplete their wealth.

Consider first the unconstrained case. From (17), it follows that the fixed indifference rate at which the environmental entrepreneur is willing to lend any amount $d_2 \in [0, h_2]$ is $\bar{r} = g$. On the other hand, the maximum contingent rate ρ which the environmental entrepreneur $i = 2$ is willing to pay for the emissions reduction (that is, the market-implied price of carbon) is $\rho = \eta$. Since $\eta > \pi - g$, the contingent rate satisfies $\rho > \pi - g$ and the standard investor $i = 1$ is willing to borrow through the contingent security and implement the green technology, with the implication that that its emissions are zero $e_1 = 0$. The environmental entrepreneur's financial returns are thus $gh_2 - \eta\frac{\theta}{1-\theta}h_1$ and they are non-negative if endowments satisfy[11]

$$h_2 \geq \frac{\eta}{g}\frac{\theta}{1-\theta}h_1. \tag{19}$$

If the investor is not willing to pay the maximum rate $\rho = \eta$, then there could be trading at the constrained rate $\rho \in [\pi - g, \eta)$ and switching to the green technology would occur if

$$h_2 \geq \frac{\pi - g}{g}\frac{\theta}{1-\theta}h_1. \tag{20}$$

However, if lenders' total endowments are such that the budget constraint (20) is violated, carbon-contingent financing at the feasible rate ρ is not enough to incentivize the technology switch for the entire population of standard entrepreneurs. In such a case, a smaller share $\theta_d \in [0, \theta)$ of standard entrepreneurs, given by $\theta_d = \frac{g(1-\theta)h_2}{(\pi-g)h_1}$, could still borrow at the limit rate $\rho = \pi - g$, and switch to the green technology g, whereas the remainder of standard entrepreneurs would continue to invest in the polluting technology π using internal finance only. The intuition behind this result is that raising external finance through the issuance of carbon-contingent debt with contingent rate $\rho < \pi - g$ does not incentivize switching to the green technology and would thus *increase* the borrowers' emissions. Environmental entrepreneurs would anticipate this and would increase the fixed interest rate at which they are willing to lend to $\bar{r} = g + \eta - \rho$.[12] It is strictly sub-optimal for standard entrepreneurs to borrow at this rate and not switch to the green technology, as they derive a higher utility from not borrowing at all and investing in their preferred polluting technology.[13]

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[11]This follows from the non-negativity constraint $g(h_2 - d_2) + \bar{r}d_2 - \rho(\bar{e}_2 - e_2) \geq 0$ with $\bar{r} = g$ and $\rho = \eta$.

[12]This follows from the utility of the entrepreneur in (17) when $e_1 = \frac{\theta}{1-\theta}d_2 + h_1$, which yields $U_2 = \max_{d_2} \bar{r}d_2 + (\rho - \eta)d_2 + g(h_2 - d_2)$.

[13]This follows from substituting $\bar{r} = g + \eta - \rho$ in standard investor's utility function (15), which yields $\pi(h_1 + d_1) - (g + \eta - \rho) - \rho d_1 - \lambda E < \pi(h_1 + d_1) - (\pi - \rho)d_1 - \rho d_1 - \lambda E = \pi h_1 - \lambda E$ given that $g + \eta > \pi$.

If the standard entrepreneur were to be a lender, then their problem would be

$$U_1 = \max_{d_1 \leq h_1} (\pi - \tau)(h_1 - d_1) + \bar{r}d_1 - \rho(\bar{e}_1 - e_1) - \lambda E, \tag{21}$$

which yields $\rho = 0$ and $\bar{r} = g$ in the presence of the tax $\tau = \pi - g$, or $\rho = 0$ and $\bar{r} = \pi$ if there is no tax $\tau = 0$. Since the standard entrepreneurs can do at least as well by investing in their preferred technology, it is optimal for them not to lend.

PROPOSITION 2. *If there is no carbon tax, a then a market for carbon-contingent financing arises in which environmental entrepreneurs act as lenders and standard entrepreneurs as borrowers. In such case*

- *if environmental entrepreneurs' endowments h_2 are sufficiently large to satisfy the inequality in* (20), *all emissions are priced at a market rate $\rho \in [\pi - g, \eta]$ and carbon-contingent debt financing enables all agents in the economy to adopt the green technology;*

- *otherwise, emissions are priced at the market rate $\rho = \pi - g$ and only a share $\theta_d = \frac{g(1-\theta)h_2}{(\pi-g)h_1} < \theta$ of standard entrepreneurs can access carbon-contingent debt financing and switch to the green technology, whereas the remainder $\theta - \theta_d$ continue to adopt the polluting technology.*

The existence of a market for carbon-contingent securities depends on whether the tax is implemented. If the carbon tax is implemented, then all emissions are priced at the tax rate $\tau = \pi - g$ and all entrepreneurs adopt the green technology, so there is no scope for pricing carbon via the financial market solution. On the other hand, if there is no tax but environmental investor's endowments h_2 are sufficiently large, all emissions are priced at a market rate $\rho \in [\pi - g, \eta]$ and contingent-debt financing enables all agents in the economy to adopt the green technology. However, if endowments are insufficiently large then emissions are priced at the market rate $\rho = \pi - g$ and only a share $\theta_d = \frac{g(1-\theta)h_2}{(\pi-g)h_1} < \theta$ of standard entrepreneurs can access contingent-debt financing and switch to the green technology, whereas the remainder $\theta - \theta_d$ continue to adopt the polluting technology.

3.4 Carbon-Contingent Financing and Political Constraints

The previous section has shown that carbon-contingent financing emerges only in the absence of the carbon tax. Borrowing through the issuance of carbon contingent securities is optimal for standard entrepreneurs, whereas lending via these securities is optimal for environmental entrepreneurs. We now take a step back and show how the possibility of being a lender (borrower) of carbon-contingent debt affects entrepreneurs' willingness to vote in favour of a carbon tax $\tau = \pi - g$, derive the constrained optimal tax and welfare in presence of financial markets, and compare it with the benchmark results outlined in Proposition 1.

If there are sufficient funds to finance the technology switch of all standard entrepreneurs, the equilibrium contingent rate lies in the region $\rho \in [\pi - g, \eta]$ and the utility of environmental lenders given the rate ρ is

$$U_2^\rho = g(h_2 - d_2) + \bar{r}d_2 - (\rho - \eta)\frac{\theta}{1 - \theta}h_1 - \lambda E, \tag{22}$$

since $\bar{e}_2 - e_2 = \frac{\theta}{1-\theta}(h_1 - e_1)$ and $e_1 = 0$. Recalling that $\bar{r} = g$ and $\rho \leq \eta$ we have that $U_2^\rho \geq U_2^\tau = gh_2$. So these agents are better off with securities rather than the tax, since their preference for contributing to reducing emissions is stronger than the price paid to incentivize standard investors to reform. On the other hand, the utility of standard borrowers that switch to the green technology

$$U_1^\rho = g(h_1 + d_1) - \bar{r}d_1 + \rho h_1 - \lambda E, \tag{23}$$

which is higher than their utility with the tax $U_1^\tau = gh_1$ because they are rewarded for reducing their emissions.

If there are insufficient funds, i.e. the budget constraint in (20) is binding, only a fraction $\theta_d < \theta$ of standard entrepreneurs can issue carbon-contingent securities priced at the minimum acceptable rate $\rho = \pi - g$. In such a case, total emissions in the economy are $E = (\theta - \theta_d)h_1$ and standard entrepreneurs have utility[14]

$$U_1^\rho = \pi h_1 - \lambda E. \tag{25}$$

These standard entrepreneurs support the tax, $U_1^\rho < U_1^\tau$, if $\tau = \pi - g < \lambda(\theta - \theta_d)$. On the other hand, the environmental entrepreneurs' utility is

$$U_2^\rho = g(h_2 - d_2) + \bar{r}d_2 - (\rho - \eta)\frac{\theta_d}{1 - \theta}h_1 - \lambda E, \tag{26}$$

which satisfies $U_2^\rho < U_2^\tau = gh_2$ if $(\eta - \rho)\frac{\theta_d}{1-\theta} < \lambda(\theta - \theta_d)$.

Note that financial markets decrease both standard and environmental entrepreneurs' support for a carbon tax $\tau = \pi - g$ by creating a more appealing counterfactual than the benchmark with internal financing only. We formalize this in the following

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[14]Note that a fraction θ_d of standard entrepreneurs have utility

$$U_1^\rho = g(h_1 + d_1) - \bar{r}d_1 + \rho h_1 - \lambda E = \pi h_1 - \lambda E \tag{24}$$

since $\bar{r} = g$ and $\rho = \pi - g$, while the remainder fraction $\theta - \theta_d$ are standard entrepreneurs that do not switch have utility $U_1^\rho = \pi h_1 - \lambda E$.

PROPOSITION 3. *Suppose that $\lambda > \pi - g$ such that the implementation of the carbon tax is desirable. If $h_2 \geq \frac{\pi-g}{g}\frac{\theta}{1-\theta}h_1$ there is never voting in favour of a carbon tax, i.e. the optimal tax that satisfies the median-voter constraint is $\tau^o = 0$. On the other hand, if $h_2 < \frac{\pi-g}{g}\frac{\theta}{1-\theta}h_1$ there are two cases*

- *if the median voter is an environmentally-oriented type $\theta < 0.5$, then either carbon emissions are taxed at $\tau^o = \pi - g$ if $\eta < \pi - g + \lambda(1-\theta)\frac{\theta-\theta_d}{\theta_d}$, or there is no carbon tax $\tau^o = 0$ and emissions are priced in financial markets using carbon-contingent securities.*

- *if the median voter is a standard type $\theta > 0.5$, then either carbon emissions are taxed at $\tau^o = \pi - g$ if $\pi - g < \lambda(\theta - \theta_d)$, or there is no carbon tax $\tau^o = 0$ and emissions are priced in financial markets using carbon-contingent securities.*

Note that if the median voter is an environmental type, the tax is no longer supported unconditionally as the global environmental benefits achieved by the tax are now traded off against the personal gains from having a greater environmental impact. Similarly, when the median voter is a standard entrepreneur, the tax threshold above which there is no support for the carbon tax is lower than it is in the baseline scenario in which carbon-contingent securities do not exist since the issuance of those securities enables them to monetize the lenders' green preferences.

Figure 5. Reducing Carbon: Carbon Tax vs Carbon-Contingent Financing

The figure shows how emission reductions are achieved with a tax or contingent finance in equilibrium as a function of the endowments of environmental entrepreneurs when the fraction of standard entrepreneurs in the population is either $\theta = 0.7$ (left plot) or $\theta = 0.3$ (right plot). The area under the black dashed line shows emission reductions when financial markets do not exist and only the politically unconstrained tax is implemented. The area under the black line represents emission reductions enabled by the optimal tax in presence of political constraints and financial markets. The relevant parameter values are $\pi/g = 1.25$, $\lambda = 1.3$, $\eta = 1.5$ and $h_1 = \$1$.
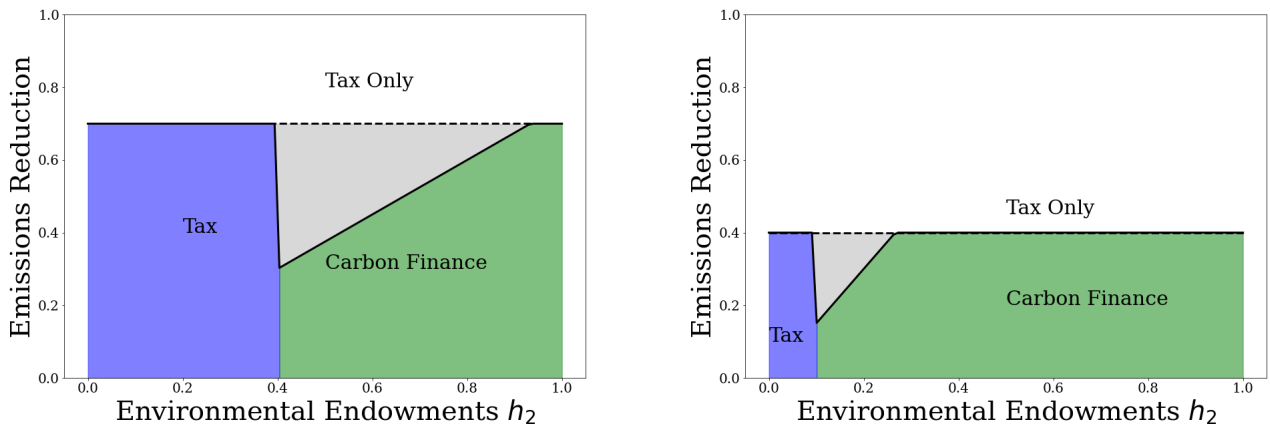
We now turn to analysing the implications in terms of carbon emissions and welfare. Figure 5 shows the equilibrium emissions reduction relative to the laissez-faire benchmark $E^* = \theta h_1$, as a function of the environmental entrepreneurs' endowments h_2 when the share of standard entrepreneurs is high $\theta = 0.7$

(left plot) and low $\theta = 0.3$ (right plot). Note first that the level of carbon emissions reduction is higher when the share of standard entrepreneurs is larger. The area under the blue line represents the emissions reduction achieved through the carbon tax in an economy without financial markets, whereas the area under the black line shows the emissions reduction delivered when financial markets exist. The blue region represents the emissions reduction achieved through the implementation of the constrained optimal tax, whereas the green region is the emissions reduction achieved through the market for carbon-contingent debt. The wedge between these two lines, shaded in grey, is the loss in emissions reduction that is essentially caused by the introduction of financial markets. Financial markets weaken the support for carbon taxes for both standard and environmental entrepreneurs and make the implementation of the tax less likely but they cannot always fund the same amount of emissions reduction that would be generated by the tax. Thus, the financial market solution is an imperfect substitute if endowments are not sufficiently large to fund the transition of all standard entrepreneurs. This imperfect substitution generates, under a set of conditions outlined in the following Proposition, welfare losses which are greater the larger the share of standard entrepreneurs. The proof is provided in the Appendix.

PROPOSITION 4. *If environmental entrepreneurs' endowments h_2 are sufficiently large to satisfy condition (19), then financial markets are always welfare improving. If endowments do not satisfy condition (20), then the implementation of the financial market solution can generate welfare losses:*

- if the median voter is an environmentally-oriented type $\theta < 0.5$, then a necessary condition for welfare loss is

$$\lambda > (\pi - g)\frac{1}{\theta_d}.$$

- if the median voter is a standard type $\theta > 0.5$, then a necessary condition for welfare loss is

$$\eta\frac{\theta_d}{\theta - \theta_d} < \lambda(1 - (\theta - \theta_d)).$$

We have therefore derived conditions under which carbon-contingent securities can substitute the carbon tax and improve welfare, and conditions under which carbon-contingent securities cannot fully substitute and generate welfare losses relative to the regulatory tool. It is worth noting that, although we have framed the security payoff in (12) as the sum of a fixed term (interest on the principal d_i), and a carbon-contingent term (difference between actual and counterfactual emissions, $e_i - \bar{e}_i$), in this stylized risk-neutral model without frictions, the role played by the former is marginal. Specifically, in equilibrium, lending any positive amount $d_2 \in (0, h_2]$ from environmental to standard entrepreneurs can occur only if the latter invest the borrowed capital in the green technology, and at an interest rate $\bar{r} = g$ which is the rate of return on the green technology. Therefore, none of the equilibrium results would change if the debt notional was normalized to $d_2 = 0$, and the environmental entrepreneurs would enter the contract at time

$t = 0$ without providing the capital upfront but only exchanging the contingent term in (12) at time $t = 1$, while continuing to invest in their preferred green technology at time $t = 0$. In the extended model with a continuum of entrepreneurs and non-linear technologies, we will make use of this property and study a simpler version of the security design where the notional d_i at time $t = 0$ is normalized to zero.

4 Extended Model

The simple model, in light of being linear delivers either-or type of predictions and cannot rationalize the empirical evidence showing that contingent finance co-exists with carbon pricing regulation. To understand the interaction between market-based and regulatory tools on the intensive margin, we extend the model to allow for a continuum of entrepreneurs with heterogeneous environmental preferences, as well as a continuum of production technologies with a convex carbon abatement cost. Specifically, instead of assuming either a polluting or a non-polluting production technology, we allow for the possibility of reducing the emission intensity of the production technology at a cost.

There is a mass one of entrepreneurs $i \in [0, 1]$ with endowments h_i, environmental preferences η_i increasing monotonically in i, and utility

$$U_i = C_i - \eta_i e_i - \lambda E, \tag{27}$$

where e_i denotes emissions associated with the actions of entrepreneur i, $E = \int_0^1 e_i di$ the total emissions in the economy, and λ is a climate parameter capturing the exposure to physical climate risk.

There is a continuum of carbon abatement technologies, parameterized by $\delta \in [0, 1]$, which for an investment input I deliver output and emissions

$$y(I, \delta) = (\pi - \phi(\delta))I \quad \text{and} \quad e(I, \delta) = I(1 - \delta),$$
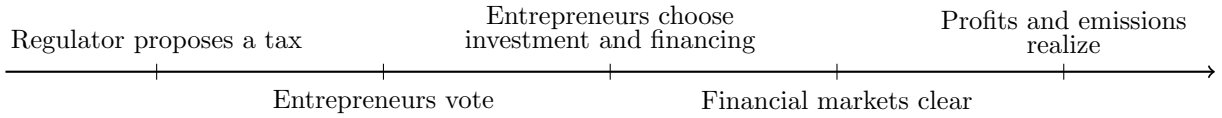
where $\phi(\delta)$ denotes the cost of abatement, assumed to be convex $\phi(\delta) = \frac{1}{2}\phi\delta^2$ and with the cost parameter satisfying $\phi > \lambda + \eta_1$.[15]

The regulator maximizes utilitarian social welfare, which is given by the total sum of utilities

$$W = \int_0^1 U_i di = \int_0^1 (C_i - \eta_i e_i) di - \lambda E.$$

The timeline below summarizes the sequence of actions in the model:

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[15]This inequality is necessary to get admissible solutions for the optimal technology δ, i.e. avoid corner solutions where the optimal technology is constrained by $\delta = 1$.

Regulator proposes a tax	Entrepreneurs choose investment and financing	Profits and emissions realize

Entrepreneurs vote   Financial markets clear

Our aim is to determine the conditions under which financial markets as a tool for pricing carbon can substitute the regulatory tool and improve welfare. To do so, we follow a backward induction approach and first determine the entrepreneurs' optimal investment and financing choices in the joint presence of a given carbon tax and a market for carbon-contingent securities. We then input those choices into the entrepreneurs' utilities at the timing of voting and derive the maximum admissible tax that each entrepreneur can support assuming each fully internalizes the behaviour of others and the adjustment of financial markets. Solving for the maximum tax as a function of the entrepreneur's type will allow us to determine the median-voter constraint, which we then input into the regulator problem of finding the constrained-optimal tax which maximizes the utilitarian social welfare in the presence of financial markets.

As a useful benchmark, we outline the investment choices, utilitarian social welfare, and cumulative emissions in a laissez-faire economy with no financial markets and no carbon taxes.

4.1 Laissez-Faire Benchmark

In a decentralized economy without financial markets nor taxes, each entrepreneur chooses investment I_i and abatement δ_i to maximize the utility in (27), where $C_i = y(I_i, \delta_i)$ and emissions $e_i = e(I_i, \delta_i)$. The investment problem reads

$$U_i^* = \max_{I_i, \delta_i} y(I_i, \delta_i) - \eta_i e(I_i, \delta_i) - \lambda E \ \text{ such that } I_i \leq h_i. \tag{28}$$

The optimal abatement choice is given by the individual environmental preference scaled by the cost of abatement

$$\delta_i^* = \frac{\eta_i}{\phi},$$

while the optimal investment given optimal abatement is

$$I_i^* = h_i \text{ if } \pi - \eta_i(1 - \frac{1}{2}\frac{\eta_i}{\phi}) > 0 \tag{29}$$

$$I_i^* = 0 \text{ otherwise.} \tag{30}$$

Assuming that the profitability of the most polluting technology π is large, we focus on the case in which condition (29) is always satisfied and it is optimal for each entrepreneur i to invest and by doing so to produce some emissions. Assuming all agents have equal endowments, each given by $h_i = \$1$, the utility

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of each agent i is

$$U_i^* = (\pi - \frac{1}{2}\frac{\eta_i^2}{\phi}) - \eta_i e_i^* - \lambda E^*, \tag{31}$$

with $e_i^* = (1 - \frac{\eta_i}{\phi})$ and total emissions $E^* = \int_i e_i^* di$. The utilitarian social welfare in this economy is given by

$$W^* = \int_0^1 U_i^* di = \int_0^1 (\pi - \eta_i + \frac{1}{2}\frac{\eta_i^2}{\phi} - \lambda(1 - \frac{\eta_i}{\phi}))di. \tag{32}$$

4.2 Carbon Tax

The regulator wants to maximize utilitarian social welfare by imposing a tax τ on the emissions e_i produced by each entrepreneur i. To preserve consistency with the previous framework where tax revenues are never effectively collected, and also motivated by extensive empirical evidence on implemented carbon tax policies, we assume that the carbon tax is *revenue-neutral*.[16] Under a revenue-neutral carbon tax, the government taxes every ton of carbon emitted and redistributes the collected tax revenues to taxpayers in the form of lump-sum payments. We assume that the redistribution rule is of a fixed type, that is, the regulator redistributes revenues as a fixed proportion α of the tax and thus makes a payment \$$\alpha\tau$ to each entrepreneur i after the revenues are collected. The design of ex-post compensations (i.e., tax rebates) is extensively studied in the context of incomplete environmental regulation and carbon leakage risk (see, for example, Martin, Muûls, De Preux, and Wagner [2014], Fowlie and Reguant [2022]). We show below that, even when considering a single economy, tax rebates have important implications on the equilibrium level of carbon-contingent financing and the voting decisions of entrepreneurs.

Consider a situation in which the entrepreneurs can finance investments with own funds only and there are no financial markets. Maintaining the assumption that the productivity of the most polluting technology π is sufficiently large so that investment is non-zero $I_i = \$1$ for each i, we have that the entrepreneur i's problem in the presence of the tax becomes

$$U_i^\tau = \max_{\delta_i} \pi - \phi(\delta_i) - \eta_i e_i(\delta_i) - \tau e_i(\delta_i) + \alpha\tau - \lambda E^\tau \tag{33}$$

with $e_i(\delta_i) = 1 - \delta_i$ and $E^\tau = \int_0^1 e_i(\delta_i^\tau)di$. The optimal abatement choice for entrepreneur i given the tax is

$$\delta_i^\tau = \delta_i^* + \frac{\tau}{\phi} = \frac{\eta_i + \tau}{\phi}, \tag{34}$$

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[16]Examples of revenue-neutral carbon taxes include both those applied to firms and those applied to consumers. As far as the former group is concerned, a popular one is the carbon tax implemented since 2001 in the United Kingdom (the Climate Change Levy). For the case of carbon taxes applied to consumers, a popular example is the tax implemented by the Canadian province of British Columbia in 2001, the first North American revenue-neutral carbon tax applied to the purchase or use of fuel in British Columbia.

which substituting into the utility (33) yields

$$U_i^\tau = \pi - \frac{1}{2}\frac{(\eta_i + \tau)^2}{\phi} - (\eta_i + \tau)(1 - \frac{\eta_i + \tau}{\phi}) + \alpha\tau - \lambda E^\tau. \tag{35}$$

Recalling that the regulator redistributes the tax revenues equally, namely

$$R^\tau = \int_0^1 \tau e_i(\delta_i^\tau)di = \alpha\tau \tag{36}$$

then one can solve for the redistribution rule

$$\alpha = 1 - \frac{\bar{\eta} + \tau}{\phi} \tag{37}$$

with $\bar{\eta} = \int_0^1 \eta_i di$ the average green preference. Substituting α into (35) yields after some re-arrangement

$$U_i^\tau = U_i^* - \frac{1}{2}\frac{\tau^2}{\phi} + \tau\frac{\eta_i - \bar{\eta}}{\phi} + \lambda(E^* - E^\tau), \tag{38}$$

with $E^* - E^\tau = \int_0^1(\delta_i^\tau - \delta_i^*)di = \frac{\tau}{\phi}$. The utility of entrepreneur i in (38) is concave in the tax τ and has a maximum in $\tau_i = \lambda + (\eta_i - \bar{\eta})$, given by the sum of the climate exposure parameter λ and a preference-specific term $(\eta_i - \bar{\eta})$, which increases monotonically in the type $i \in [0, 1]$. By comparing the utility with the tax in (38) with the laissez-faire utility in (31), we can derive the maximum acceptable tax

$$\tau \leq \bar{\tau}_i = 2\tau_i. \tag{39}$$

Any tax above the threshold $\bar{\tau}_i$ makes the entrepreneur strictly worse off than the laissez-faire benchmark, i.e. $U_i^\tau - U_i^* < 0$. In this economy where financial markets are not taken into account, we show in the Appendix that utilitarian social welfare is

$$W^\tau = W^* - \frac{1}{2}\frac{\tau^2}{\phi} + \lambda(E^* - E^\tau) \tag{40}$$

The regulator chooses the optimal tax by maximizing the utilitarian social welfare in 40 subject to the constraint that the chosen tax is below the threshold of the median voter $\bar{\tau}_{0.5}$.

PROPOSITION 5. *For a given median-voter constraint $\bar{\tau}_{0.5}$ as in (39), the tax τ^o that maximizes the regulator problem*

$$\max_\tau W^\tau \quad such\ that \quad \tau \leq \bar{\tau}_{0.5} \tag{41}$$

with utilitarian social welfare as in (40) is given by $\tau^o = \min(\lambda, \bar{\tau}_{0.5})$.

The proposition shows that the unconstrained optimal tax equates the Pigouvian benchmark given by the parameter capturing exposure to climate shocks, i.e. $\tau^o = \lambda$. However, when the political constraint is binding the optimal tax is the one that makes the median voter indifferent between supporting or not the regulation, namely $\tau^o = 2\lambda + 2(\eta_{0.5} - \bar{\eta})$. Note that the lower is the green preference of the median voter relative to the average, the lower is the admissible tax threshold and the more likely it is that the political constraint will bind i.e. the more stringent the political constraint. If preferences are uniformly distributed across types, the median voter has the average green preference $\eta_{0.5} = \bar{\eta}$ and the regulator can always enforce the unconstrained optimum.[17] We will return to this solution as a corner case for the regulator problem in the presence of financial markets.

4.3 Carbon-Contingent Financing

Given a certain carbon tax τ, we derive the conditions under which a market for carbon-contingent financing exists and the equilibrium price of carbon implied by this market. We introduce carbon-contingent securities along the lines of those studied in the previous baseline model. Specifically, we assume that each entrepreneur i can issue a carbon-contingent security which effectively rewards the issuer for reducing emissions and imposes a monetary penalty for increasing emissions relative to a benchmark agreed at security issuance. Without loss of generality, we assume a zero principal notional at time $t = 0$ and focus on the carbon-contingent part of the security payoff.[18] Under this simplified security structure, the payoff to the security issuer at time $t = 1$ is given by

$$\rho(e_i^\tau - e_i), \tag{42}$$

where e_i the issuer i's actual emissions at time $t = 1$ and e_i^τ the benchmark emissions given by the counterfactual scenario where the security is not issued and there is a carbon tax τ, determined at time $t = 0$. Thus, the security issuer is rewarded with a positive payoff if it reduces emissions below the benchmark, and vice-versa.

We first derive the issuer and lender problem and then outline the conditions under which the net gains from issuing the security are a monotonically decreasing function of the type. We then solve for a cutoff type which is indifferent between selling or buying a carbon-contingent security, and derive the equilibrium price of carbon emissions implied by the contract, ρ, as a function of this type. The equilibrium will allow us to determine the financial market response to the tax τ, which we will then input into the regulator's problem.

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[17]This relies on the assumption of equal endowments $h_i = \$1$, as well as on the choice of the tax redistribution rule.

[18]Note that for simplicity of the analysis, and following the discussion in the previous section, we have normalized the notional d_i in (12) to zero and decided to only focus on the equilibrium pricing of the contingent term of the security. The security could also be interpreted as a carbon swap which has zero price at issuance and an exchange of a variable component ρe_i for a fixed component $-\rho\bar{e}_i$ at time $t = 1$.

The Issuer's Problem. Denote $\mathcal{I} \subset [0,1]$ the set of entrepreneurs that issue the carbon-contingent contract and thus act as sellers in this market. Denote $\mathcal{I}_i^\tau(\rho)$ issuer i's utility for a given tax τ and security-implied carbon price ρ, which is given by

$$\mathcal{I}_i^\tau(\rho) = \max_{\delta_i} U_i^\tau(\delta_i) + \rho(e_i^\tau - e_i(\delta_i)) \text{ such that } \delta_i \leq 1 \tag{43}$$

with utility under the tax $U_i^\tau(\delta_i) = \pi - \phi(\delta_i) - \eta_i e_i(\delta_i) - \tau e_i(\delta_i) + \tau\alpha - \lambda E^\tau$, emissions $e_i(\delta_i) = 1 - \delta_i$, and subject to the technology constraint that the maximum abatement is one which reduces emissions to zero. The optimal carbon abatement choice is

$$\delta_i^\tau(\rho) = \delta_i^\tau + \frac{\rho}{\phi} \text{ if } \eta_i < \phi - \tau - \rho$$
$$\delta_i^\tau(\rho) = 1 \qquad \text{otherwise,} \tag{44}$$

where $\delta_i^\tau = \frac{\eta_i + \tau}{\phi}$ is the optimal abatement technology choice in the counterfactual scenario where the security is not issued, derived in (34). Substituting the optimal technology back into the utility in (43), we have

$$\mathcal{I}_i^\tau(\rho) = U_i^\tau(\delta_i^\tau) + \frac{1}{2}\frac{\rho^2}{\phi}\mathbb{1}\{\eta_i < \phi - \tau - \rho\}, \tag{45}$$

so issuing a carbon-contingent security yields strictly positive profits with respect to a benchmark utility with the carbon tax only, as long as $\rho \in (0, \phi - \tau - \eta_i)$. Note that the lower the entrepreneur's green preference η_i, the more likely it is that the entrepreneur will benefit from the security issuance.

The Lender's Problem. Denote now the set of lenders, which act as buyers of carbon-contingent contracts, as $\mathcal{L} \subset [0,1] - \mathcal{I}$. Denote the total quantity of contracts, i.e. emissions reduction, purchased by entrepreneur $i \in \mathcal{L}$ as

$$q_i = \int_{j \in \mathcal{I}_i} (e_j^\tau - e_j) dj, \tag{46}$$

with $\mathcal{I}_i$ the set of issuers whose contingent securities are purchased by i, with $\int_{i \in \mathcal{L}} \mathcal{I}_i = \mathcal{I}$. Entrepreneur i continues to invest in the abatement technology δ_i^τ, and only decides the optimal quantity q_i of carbon-contingent contracts to purchase, by solving the problem

$$\mathcal{L}_i^\tau(\rho) = \max_{q_i} U_i^\tau(\delta_i^\tau) + \eta_i q_i - \rho q_i \text{ such that } \pi_i^\tau(\rho) - \rho q_i \geq 0, \tag{47}$$

where $U_i^\tau(\delta_i^\tau)$ is the benchmark utility in the presence of the carbon tax, $\pi_i^\tau(\rho) = \pi - \frac{1}{2}\frac{\eta_i^2}{\phi} - \frac{1}{2}\frac{\tau^2}{\phi} - \frac{\tau(\bar{\eta} + \rho\int_{i \in \mathcal{I}} di)}{\phi}$ is the financial return from investing in the technology after the tax, whose derivation is provided in the Appendix, and the constraint is the equivalent of the budget constraint introduced in the

previous section. From the linearity of the problem, it follows that

$$q_i^\tau(\rho) = \frac{\pi_i^\tau(\rho)}{\rho} \quad \text{if} \quad \rho \leq \eta_i$$

$$q_i^\tau(\rho) = 0 \quad \text{otherwise.}$$

(48)

Substituting the optimal quantity back into the utility in (47), the utility of lender i given the tax τ and the security-implied carbon price ρ is given by

$$\mathcal{L}_i^\tau(\rho) = U_i^\tau(\delta_i^\tau) + (\eta_i - \rho)\frac{\pi_i^\tau(\rho)}{\rho}1\{\eta_i \geq \rho\},$$

(49)

meaning that entrepreneur i is strictly better off purchasing the security if $\eta_i > \rho$, realizing profits that depend on the net return on the technology $\pi_i^\tau(\rho)$, otherwise it has the same utility as in the benchmark scenario where the security is not issued.

Define the net gains from issuing the security as the difference between the entrepreneur i's utility associated with issuing a carbon-contingent security, given in (45), and the utility associated with acting as a lender in carbon-contingent security markets, given in (49). The net gains from issuing the carbon-contingent security are

$$\Pi_i^\tau(\rho) = \mathcal{I}_i^\tau(\rho) - \mathcal{L}_i^\tau(\rho) = \frac{1}{2}\frac{\rho^2}{\phi}1\{\eta_i < \phi - \tau - \rho\} - (\eta_i - \rho)\frac{\pi_i^\tau(\rho)}{\rho}1\{\eta_i > \rho\},$$

(50)

and are decreasing with the type i on the extensive margin. Proposition 6 below outlines sufficient conditions for the profits in (50) to decrease monotonically with the type i for each $i \in [0,1]$ given the set of issuers $\mathcal{I} = [0,i)$ and the set of lenders $\mathcal{L} = [i,1]$.

PROPOSITION 6. *For a given abatement cost ϕ, profitability π, carbon tax τ and green preferences $\eta_i \in \mathcal{C}^1([0,1])$ with $\eta_i' > 0$, a sufficient condition for the net gains in (50) to decrease monotonically with the type i is that*

$$\pi > \frac{3}{2}\frac{\eta_i^2}{\phi} + \frac{\eta_i\rho^2}{\phi\eta_i'} + \frac{1}{2}\frac{\tau^2}{\phi} + \frac{\tau(\bar{\eta} + \rho i)}{\phi}$$

(51)

with $\bar{\eta} = \int_0^1 \eta_i di$ the average green preference.

The proof is provided in the Appendix. When the single-crossing property is verified, we can solve for an internal cutoff type $x \in (0,1)$ verifying $\Pi_x^\tau(\rho) = 0$ such that the set of issuers is $\mathcal{I} = [0,x)$ and the set of lenders is $\mathcal{L} = [x,1]$. The higher the cutoff type x the higher the fraction of agents that are borrowing through carbon-contingent securities and the higher the level of contingent financing in the economy. Formally, we introduce the following

Definition. *The pair (ρ, x) constitutes an equilibrium if*

a) the market clearing condition

$$\int_0^x (e_i^\tau - e_i(\delta_i^\tau(\rho)))di = \int_x^1 q_i^\tau(\rho)di \qquad (52)$$

is satisfied, with $\delta_i^\tau(\rho)$ as in (44) and $q_i^\tau(\rho)$ the optimal quantity purchased as in (48), and

b) the indifference condition

$$\Pi_x^\tau(\rho) = 0 \qquad (53)$$

is satisfied, with $\Pi_i^\tau(\rho)$ the net gains in (50) for $i = x$.

We can therefore outline the following.

PROPOSITION 7. *For a given abatement cost ϕ, profitability π, carbon tax τ, green preferences $\eta_i \in \mathcal{C}^1([0,1])$ with $\eta_i' > 0$ which satisfy conditions (51), the pair (ρ, x) which solves*

$$\rho = \frac{-\tau(1-x) + \sqrt{(\tau(1-x))^2 + 4\phi k(x)}}{2} \quad and \quad \frac{1}{2}\frac{\rho^2}{\phi} = (\eta_x - \rho)\frac{\pi_x^\tau(\rho)}{\rho} \qquad (54)$$

with $k(x) = \frac{1}{x}(\pi - \frac{1}{2}\frac{\tau^2}{\phi} - \frac{\tau\bar\eta}{\phi})(1-x) - \frac{1}{x}\int_x^1 \frac{1}{2}\frac{\eta_i^2}{\phi}di$, constitutes an equilibrium if $\eta_x < \phi - \tau - \rho$.

Otherwise, a corner solution exists with $\rho = \eta_x = \frac{\phi - \tau}{2}$ and technology-constrained market clearing $\int_0^x (e_i^\tau - e_i^\tau(\delta_i^\tau(\rho)))di = (1 - \frac{1}{2}\frac{\tau}{\phi})x < \int_x^1 q_i^\tau(\rho)di$.

Figure 6. Equilibrium carbon-contingent financing as a function of the tax

The plots show the equilibrium rate ρ (left plot) and the cutoff type x (right plot) in (54) as a function of the tax τ when preferences are either convex $\eta_i = \eta i^2$ (black line) or concave $\eta_i = \eta\sqrt{i}$ (blue line) in the type $i \in [0,1]$. Endowments are $h_i = \$1$ for each i, and the other model parameters are $\eta = 1$, $\phi = 5$, and $\pi = 3$.
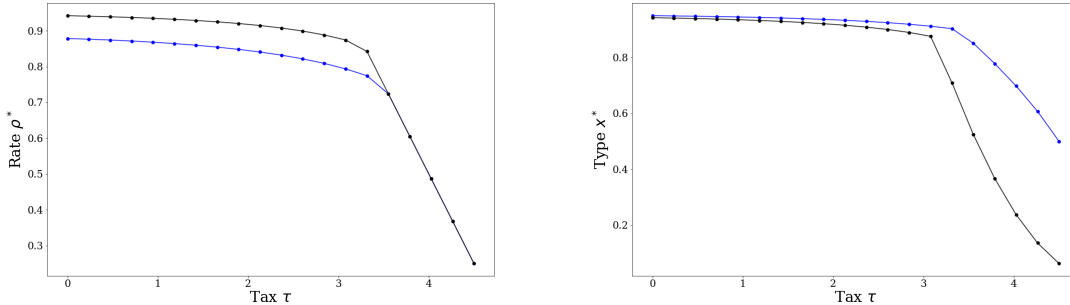

Figure 6 shows the equilibrium carbon rate ρ (left plot) and the cutoff type x (right plot) against the

tax τ. Preferences are assumed to be convex $\eta_i = \eta i^2$ (black line) or concave $\eta_i = \eta\sqrt{i}$ (blue line) in the entrepreneur type $i \in [0,1]$, with the preference of the highest type $\eta = \$1/CO_2$. As observed, the equilibrium rate ρ, which represents the market-implied price of carbon expressed in $\$/CO_2$, as well as the cutoff type x, are a piece-wise decreasing function of the tax τ. This implies that the equilibrium carbon abatement financed through carbon-contingent securities is also a decreasing function of the tax τ.[19] The negative effect of the tax on the price and level of carbon contingent financing is twofold. On the demand side, the regulatory compliance costs associated with the carbon tax reduce the entrepreneurs' budget, thereby reducing their lending capacity. Importantly, the effect is a function, among other things, of the ex-post tax redistribution rule chosen by the regulator, suggesting that different designs of tax rebates alter the utilitarian social welfare of the regulated economy even in absence of carbon leakage risk. On the supply side, a higher carbon tax reduces the abatement potential of the security issuers by simply reducing the number of technologies available to reduce emissions beyond the counterfactual benchmark. The supply effect is much stronger than the demand one, but is present for large values of the tax τ only (e.g., when the tax is more than three times higher than the green preference of the highest type $i = 1$, which is depicted on the right side of Figure 6). Before the supply effect kicks in, it is useful to note that the cutoff type which is indifferent between being a lender or an issuer of carbon-contingent securities is well above the median type $i = 0.5$, meaning that the median type is typically an issuer of carbon-contingent securities.

4.4 The Voting Problem

We now solve for the entrepreneurs' voting problem and determine the maximum admissible tax that a regulator can enforce without loosing political support from the majority. We do so by taking into account that the existence of financial markets for pricing carbon affects political support for regulation. Since preferences for the tax τ increase monotonically with the type $i \in [0,1]$, as we show below, the maximum admissible tax is the one that makes the median type $i = 0.5$ indifferent between voting or not for the carbon tax.[20] As discussed in the previous section, the median voter type is likely to be an issuer of carbon contingent securities, unless the tax τ is so large that the technology constraint is binding i.e. $\delta_i = 1$ such that $e_i(1) = 0$. In what follows, we focus on the case in which the magnitude of the tax is comparable to the green preference of the highest type, which allows us to abstract from corner solutions in which the equilibrium relationship between the tax and the price of carbon-contingent securities is determined by the technology constraint.

To assess support for regulation, we contrast the median voter's utility in an economy with carbon-contingent financing and no taxes, against one with carbon-contingent financing and taxes. Define $\mathcal{I}_i^*$

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[19]This follows from $\int_0^x (e_i^\tau - e_i^\tau(\delta_i^\tau(\rho)))di = \int_0^x (\frac{\rho}{\phi})di = \frac{\rho x}{\phi}$.
[20]This is because we assume that endowments are equally distributed across types $i \in [0,1]$.

issuer i's utility in a laissez-faire economy with financial markets and no carbon tax, and denote the equilibrium price of the carbon-contingent security in this economy as ρ^*. Define $\mathcal{I}_i^\tau$ as the utility in an economy with financial markets and a carbon tax, with (ρ, x) the equilibrium price and level of contingent financing as in (54). We show in the Appendix that issuer i's utility gain from regulation, as a function of the tax τ, can be written as

$$\mathcal{I}_i^\tau - \mathcal{I}_i^* = -\frac{1}{2}\frac{\tau^2}{\phi} + \tau\frac{\eta_i - \bar{\eta} - \rho x}{\phi} + \lambda(E^* - E^\tau) - \frac{1}{2}\frac{\rho^{*2} - \rho^2}{\phi} \tag{55}$$

with $E^* - E^\tau$ the excess abatement generated by the tax against the laissez-faire benchmark with financial markets only. The net utility from voting in favour of a given tax τ is a decreasing function of the tax and the voting condition can be expressed as

$$\mathcal{I}_i^\tau - \mathcal{I}_i^* \geq 0 \ \ \text{iff} \ \ \tau \leq \bar{\tau}_i \tag{56}$$

with $\bar{\tau}_i$ a type-specific threshold given by

$$\bar{\tau}_i = \tau_i + \sqrt{(\tau_i)^2 - ((\rho^* - \rho)^2 + 2\lambda(\rho^* x^* - \rho x))} \tag{57}$$

with $\tau_i = \lambda - \rho x + (\eta_i - \bar{\eta})$ the tax that optimizes the utility in (55) when the marginal effect of the tax τ on the price and level of carbon contingent financing is negligible, and $(\rho^* - \rho)^2 + 2\lambda(\rho^* x^* - \rho x)$ the term accounting for the equilibrium adjustment of the security-implied carbon price and the total emissions abatement brought about by carbon-contingent securities. The derivation of (57) is provided in the Appendix.

Let us compare the threshold which accounts for the equilibrium implications of financial markets for pricing carbon, given in (57), with that obtained when financial markets are not taken into account, which as shown in (39) is given by $\bar{\tau}_i = 2\tau_i = 2(\lambda + (\eta_i - \bar{\eta}))$. In both cases, support for the carbon tax increases monotonically with the type i. The tax threshold is lower the lower the green preference of the median voter $\eta_{0.5}$ relative to the average green preference $\bar{\eta}$, with the implication that the political constraint is more stringent or more likely to bind. However, we note that the threshold is lower when financial markets are taken into account for two reasons. The first is that the presence of financial markets decreases the carbon tax revenues, thereby reducing the ex-post lump-sum transfer to each entrepreneur. This is captured by the term ρx, which decreases τ_i relative to the baseline case without financial markets. The second reason is that the tax makes the price of the carbon implied by the contingent security decrease (i.e. $\rho^* > \rho$), thereby reducing the profits from the issuance of a carbon contingent security. Given that the threshold is lower, financial markets reduce the probability of a given tax τ being implemented. We now solve for the regulator problem, outline the constrained-optimal tax, and discuss under what conditions

the presence of financial markets generate lower (higher) abatement than the regulatory tool alone, and the implications in terms of utilitarian social welfare.

4.5 The Regulator Problem

As in the baseline model, the regulator is subject to a political constraint in that it must propose a tax which is supported by at least half of the population. The regulator utilitarian welfare in the presence of financial markets is

$$W^\tau(\rho) = \int_0^x \mathcal{I}_i^\tau(\rho)di + \int_x^1 \mathcal{L}_i^\tau(\rho)di \tag{58}$$

where $I_i^\tau(\rho)$ and $\mathcal{L}_i^\tau(\rho)$ are the utilities of the borrowers and lenders in (45) and (47), respectively, evaluated at the equilibrium carbon price ρ, and x is the equilibrium indifference type which pins down the level of contingent financing. As discussed, we limit the analysis to the case in which the equilibrium pair (ρ, x) satisfies the interior condition in (54), which amounts to assuming that the tax τ is not too large and there there are always technologies available to further increase abatement. In such a case, we prove in the Appendix the following

PROPOSITION 8. *For a given threshold $\bar{\tau}_{0.5}$ in (57), the optimal tax τ^o which maximizes the constrained regulator problem*

$$\max_\tau W^\tau(\rho) \quad \text{such that} \quad \tau \leq \bar{\tau}_{0.5}$$

with utilitarian social welfare in (58) satisfies

$$\tau^o = \min\left(\lambda - \frac{\rho^o \rho_\tau^o x^o + \frac{1}{2}(\rho^o)^2 x_\tau^o + \rho^o x^o - \phi f_\tau^o}{1 + \rho_\tau^o x^o + x_\tau^o \rho^o}, \bar{\tau}_{0.5}\right) \tag{59}$$

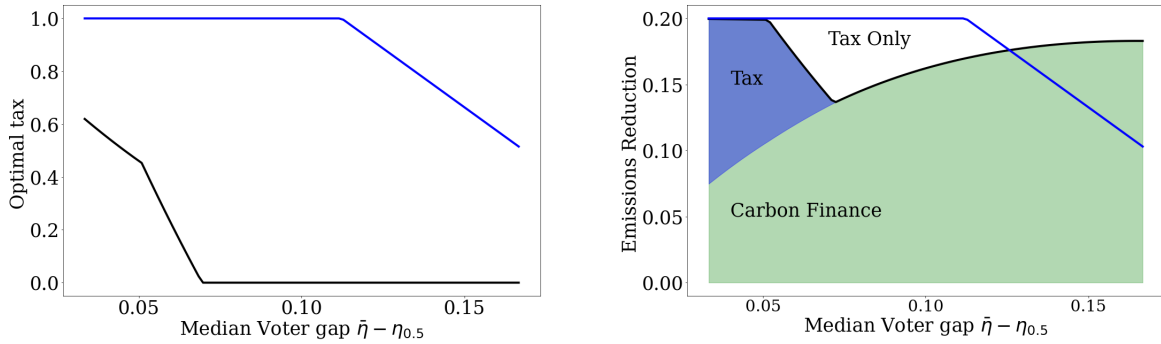*with (ρ^o, x^o) the equilibrium pair in (54) evaluated at the tax $\tau = \tau^o$, (ρ_τ^o, x_τ^o) the derivative of (ρ^o, x^o) in (54) with respect to τ evaluated at $\tau = \tau^o$, and $f_\tau^o = \frac{\partial}{\partial \tau}\int_x^1 \frac{\pi_i^\tau(\rho)}{\rho}di$ evaluated at $\tau = \tau^o$. If the median-voter constraint is not binding, the welfare in (58) given the optimal tax $W^{\tau^o}(\rho^o)$ is higher than the welfare achieved by the implementation of a Pigouvian carbon tax, $\tau^o = \lambda$, only.*

The left plot in Figure 7 shows, in black, the constrained optimal tax τ^o in (59) as a function of the difference between the average and median voter's preference, $\bar{\eta} - \eta_{0.5}$, further referred to as the median voter gap. The higher the deviation of the median voter's environmental preference from the average, the lower the admissible tax threshold and the more stringent the political constraint is said to be. The blue line depicts the the benchmark optimal tax derived in the absence of financial markets outlined in Proposition 5. The right plot in Figure 7 shows, under the black line, the emissions reduction achieved by the combined presence of financial markets and the optimal tax τ^o in (59), with the relative contribution of the two

tools being represented by the blue region and the green region, respectively. The emissions reduction is compared to that achieved by the use of the carbon tax in (41), represented by the area under the blue line.

Figure 7. Reducing Carbon: Carbon Tax vs Carbon-Contingent Financing

The left plot shows the optimal tax τ^o against the median voter gap $\bar{\eta} - \eta_{0.5}$. The blue line represents the baseline case where financial markets are not present. The black line plots the optimal tax with financial markets. The right plot shows the emission abatement in excess of the laissez-faire benchmark in which both the carbon tax and financial markets are absent. The green region is the amount of emission abatement achieved through issuance of carbon-contingent securities. The blue region is the residual abatement achieved through the tax. The area below the blue line represents the baseline emissions abatement achieved by the tax only, i.e. when financial markets are not present. Preferences are right-skewed with $\eta_i = 0$ for $i \in [0, .5)$ and $\eta_i = \eta i$ for $i \in [.5, 1]$. Other model parameters are $\phi = 5$, $\lambda = 1$, $\pi = 3$.

Let us first consider the unconstrained scenario, that is, a scenario in which the median voter preference $\eta_{0.5}$ is high enough, and the median voter gap low enough, that the optimal tax can always be implemented (left region of left plot in Figure 7). Note that if financial markets are not taken into account when voting, the optimal carbon tax equates the Pigouvian benchmark $\tau^o = \lambda$, as the blue line in Figure (7) shows. On the other hand, when the financial market response is taken into account (black line in Figure 7, left plot), the tax is lower than the Pigouvian benchmark by an optimal amount that depends on the emissions abatement enabled by financial markets and the marginal effect that the tax τ has on such abatement in equilibrium. Since the marginal effect of the tax on the security-implied carbon price and the cutoff type is small, i.e. the derivative of ρ and x are small, we show in the Appendix that the unconstrained optimal tax can be well approximated by

$$\tau^o = \lambda - \rho^o x^o, \tag{60}$$

meaning that the optimal tax τ^o differs from the Pigouvian benchmark by an amount that perfectly offsets the abatement enabled by financial markets. In such a scenario, there is perfect substitution between the regulatory and the financial market tool as far as the final amount of emissions reduction is concerned. As illustrated on the leftmost side of the right plot in Figure 7, the abatement achieved by the use of the regulatory tool only is the same as the one achieved using a combination of financial market and regulatory tools. However, as stated in Proposition 8, the optimal welfare in presence of financial markets

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is strictly above the benchmark case in which only the tax is implemented. This is because the markets allow green entrepreneurs to optimally increase their environmental impact by financing the emissions reduction of standard entrepreneurs.

We therefore consider the intermediate case in which green preferences of the median voter are i) low enough to make the political constraint in the presence of financial markets binding, but also ii) high enough to maintain the political constraint in the absence of financial markets slack (intermediate region in Figure 7, left plot).[21] This is an intriguing case that suggests that the presence of financial markets reduces the regulator's ability to implement the unconstrained optimal tax. In such a scenario, the optimal tax when financial markets do not exist is optimally set to the Pigouvian benchmark, but it is zero if the presence of financial markets is internalized when voting (intermediate region in Figure 7, left plot). Consequently, the emissions reduction achieved by the carbon tax alone is higher than the one achieved by the combined presence of the carbon tax and the financial markets (intermediate region in Figure 7, right plot). Whether the loss in emissions reduction generated by the introduction of financial markets translates into a welfare loss is ambiguous and depends on the remainder of model parameters, as the reduction in welfare due to a higher pollution externality is partially or fully offset by an increase in the green entrepreneurs' utility brought about by the carbon-contingent security purchase. This case is comparable to the cases discussed in the linear model where financial markets make the tax less likely to be implemented while not being able to fully substitute the regulatory tool in terms of the desired level of emissions reduction.

Finally, we consider the case in which the median-voter's preference is so low that the political constraint is binding independently of the presence of financial markets (rightmost region of left plot in Figure 7). In this case, the tax is sub-optimally below the Pigouvian benchmark and can never achieve the desired level of emissions reduction, whereas financial markets can achieve a higher level of emissions reduction given a homogeneous distribution of endowments. Similarly to the baseline model, financial markets increase the equilibrium level of emissions reduction resulting in a welfare gain with respect to the benchmark economy in which there is a carbon tax only.

5   Concluding Remarks

We start by proposing a baseline model in which financially- and environmentally- motivated entrepreneurs can invest their endowments in polluting or non-polluting technologies, with the latter being less profitable than the former. We show that a carbon tax corrects the laissez-faire allocation in which the polluting

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[21]This can happen because, as discussed in the previous section, the presence of financial markets makes the regulation *less* appealing for the median voter.

technology is adopted by standard entrepreneurs, and has the effect of increasing welfare and decreasing emissions. If there is no political support for a carbon tax, carbon-contingent financing provided by environmentally-motivated entrepreneurs can effectively substitute the carbon tax. Whether the financial market solution partially or fully substitutes regulation depends importantly on the endowments of environmental entrepreneurs who, by lending to financially-motivated entrepreneurs via carbon-contingent contracts, are essentially subsidizing their investment in the non-polluting technology. We show that when environmental entrepreneurs are endowed with sufficiently large funds, financial markets are a superior alternative to the regulatory tool in that they achieve higher welfare than the carbon tax alone independently of the stringency of the political constraint. Pricing emissions through financial markets creates welfare gains also when environmental funds are small, provided that the political constraint is also binding. However, when financial markets shift an economy from one that supports a carbon tax to one that does not, and the capital deployed through carbon-contingent financing is small, there can be welfare losses.

We then extend the model to a continuum of entrepreneurs with heterogeneous environmental preferences and production technologies the emission intensity of which can be reduced at a convex abatement cost. We derive the optimal tax when the regulator is politically constrained in implementing a revenue-neutral carbon tax which involves redistributing the revenues from the tax across voters. Solving for the entrepreneurs' financing and investment decisions while taking account of the financial market's response to the tax, we show that taxation and carbon-contingent financing can co-exist and derive the conditions under which together they achieve higher welfare than the carbon tax alone. They are still characterized by a substitution relationship, and the share of emission reduction enabled through carbon-contingent financing is smaller the higher the tax, suggesting that such capital flows are best directed to unregulated markets where they can have more impact.

A natural next step is to endogenize the choice of the tax revenue redistribution rule, as the latter has an impact on the equilibrium level of carbon contingent financing and therefore welfare. The welfare impact of ex-post compensations has been extensively studied in the context of emissions leakage and competitiveness concerns, but to the best of our knowledge have never been studied in the context of the interaction between regulation and financial markets. Recognizing that financing the climate transition requires significant investments and involves highly uncertain variables and complex economy-climate interactions [Stern, Stiglitz, and Taylor, 2022], we also aim to extend the analysis to allow for uncertainty and financial constraints. Finally, it would be interesting to extend the analysis to account for the fact that carbon taxes are not the only regulatory tool for reducing carbon, and focus specifically on cap-and-trade carbon emission trading schemes. Carbon taxes are price tools which involve directly placing a price on carbon, with quantities of carbon use and emissions adjusting in response. Cap-and-trade markets are quantity instruments which involve constraining the quantity of carbon entering the economy

through emission allowances, with prices emerging indirectly from the market for allowances. While a clear distinction between these two is the market-determined, uncertain carbon price associated with the cap-and-trade system, these have been argued to be more similar than different [Goulder and Schein, 2013, Stavins, 2020]. Understanding the features that make them equivalent, how they can be optimally combined and how they interact with the financial market solution we study in this paper, as well as alternative financial market tools and security designs such as a carbon swap, are important avenues for future research.

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A   Appendix

Proof. [Proposition 4] If the median voter is an environmental type $\theta < 0.5$, then the tax passes in the counterfactual scenario when financial markets are not internalized upon voting but an environmental investor that anticipates the effect of carbon-contingent financing will not support the tax if $\eta - (\pi - g) > \lambda \frac{\theta - \theta_d}{\theta_d}(1 - \theta)$. Welfare with carbon-contingent financings

$$W^\rho = [\pi\theta + (\eta - \pi + g)\theta_d]h_1 + (1 - \theta)gh_2 - \lambda(\theta - \theta_d)h_1, \tag{61}$$

while welfare with the tax is $W^\tau = \theta g h_1 + (1 - \theta)gh_2$, so the difference in welfare is

$$W^\rho - W^\tau = [\eta - (\pi - g)]\theta_d h_1 + (\pi - g)\theta h_1 - \lambda(\theta - \theta_d)h_1. \tag{62}$$

So we have that $W^\rho - W^\tau < 0$ if

$$\eta - (\pi - g) < \lambda\frac{\theta - \theta_d}{\theta_d} - (\pi - g)\frac{\theta}{\theta_d}.$$

Conditions for welfare loss are

$$\eta - (\pi - g) < \lambda\frac{\theta - \theta_d}{\theta_d} - (\pi - g)\frac{\theta}{\theta_d} \quad \text{and} \quad \eta - (\pi - g) > \lambda\frac{\theta - \theta_d}{\theta_d}(1 - \theta).$$

The parameter region is not empty if

$$\lambda\frac{\theta - \theta_d}{\theta_d}(1 - \theta) < \lambda\frac{\theta - \theta_d}{\theta_d} - (\pi - g)\frac{\theta}{\theta_d}$$

$$\lambda > (\pi - g)\frac{1}{\theta_d}$$

If the median voter is standard entrepreneur $\theta > 0.5$, then the tax passes in the counterfactual scenario when financial markets are not present if $\pi - g < \lambda\theta$. However, when the voter anticipates presence of financial markets, the tax passes only if $\pi - g < \lambda(\theta - \theta_d)$ so the welfare implications with and without financial markets are different if $\lambda(\theta - \theta_d) < \pi - g < \lambda\theta$.

The conditions under which the presence of financial markets can cause welfare losses are

$$\eta - (\pi - g) < \lambda\frac{\theta - \theta_d}{\theta_d} - (\pi - g)\frac{\theta}{\theta_d} \quad \text{and} \quad \lambda(\theta - \theta_d) < \pi - g < \lambda\theta$$

which can be re-written as

$$(\pi - g) < \lambda - \eta\frac{\theta_d}{\theta - \theta_d} \quad \text{and} \quad \lambda(\theta - \theta_d) < \pi - g < \lambda\theta$$

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which means

$$\lambda(\theta - \theta_d) < \pi - g < min(\lambda\theta, \lambda - \eta\frac{\theta_d}{\theta - \theta_d})$$

If $\lambda\theta < \lambda - \eta\frac{\theta_d}{\theta - \theta_d}$, which can be re-written as $\eta\frac{\theta_d}{\theta - \theta_d} < \lambda(1 - \theta)$, then the region is non empty if

$$\lambda(\theta - \theta_d) < \lambda\theta,$$

which holds since $\theta_d < \theta$. Otherwise it must hold that $\lambda(\theta - \theta_d) < \lambda - \eta\frac{\theta_d}{\theta - \theta_d}$, which means

$$\eta\frac{\theta_d}{\theta - \theta_d} < \lambda(1 - \theta + \theta_d).$$

Note that this condition is less stringent than $\eta\frac{\theta_d}{\theta - \theta_d} < \lambda(1 - \theta)$.

Proof. [Proposition 5] The utilitarian social welfare is given by

$$W^\tau = \int_0^1 (U_i^* - \frac{1}{2}\frac{\tau^2}{\phi} + (\eta_i - \bar\eta)\frac{\tau}{\phi})di + \lambda(E^* - E^\tau) \tag{63}$$

with $E^* - E^\tau = \frac{\tau}{\phi}$ and $\int_0^1 \eta_i = \bar\eta$. It is therefore immediate to note that

$$\frac{d^2 W^\tau}{d\tau^2} < 0 \quad \text{and} \quad \frac{dW^\tau}{d\tau}\Big|_{\tau=\lambda} = 0 \tag{64}$$

from which follows that there is a unique unconstrained optimum in $\tau^o = \lambda$ and $W^\tau > W^*$ for $\tau \in (0, \lambda]$.

Proof. [Proposition 6] The net gains from issuance of the carbon-contingent security are

$$\Pi_i^\tau(\rho) = \frac{1}{2}\frac{\rho^2}{\phi}1\{\eta_i < \phi - \tau - \rho\} - (\eta_i - \rho)\frac{\pi_i^\tau(\rho)}{\rho}1\{\eta_i > \rho\}. \tag{65}$$

We want to prove that

$$\frac{\partial}{\partial i}\Pi_i^\tau(\rho) < 0 \tag{66}$$

for each $i \in [0, 1]$. The first term in (65) is a decreasing step function of the type η_i equal to $\frac{1}{2}\rho^2/\phi$ for $\eta_i < \phi - \tau - \rho$ and equal to zero for $\eta_i \geq \phi - \tau - \rho$. On the other hand, the second term in (65) is equal to zero if $\eta_i < \rho$ and equal to $-(\eta_i - \rho)\frac{\pi_i^\tau(\rho)}{\rho}$ for $\eta_i > \rho$. To prove (66), it is therefore sufficient to prove that

$$\frac{d}{di}\left((\frac{\eta_i}{\rho} - 1)\pi_i^\tau(\rho)\right) > 0. \tag{67}$$

The financial return of the lender $\pi_i^\tau(\rho)$ is the return of the technology after the tax payment has been made

$$\pi_i^\tau(\rho) = \pi - \phi(\delta_i^\tau) - \tau e_i^\tau + \tau\alpha^\tau(\rho). \tag{68}$$

Recalling that $\delta_i^\tau = \frac{\eta_i + \tau}{\phi}$, emissions $e_i^\tau = 1 - \delta_i^\tau$, and $\alpha^\tau(\rho) = 1 - \frac{\bar{\eta}}{\phi} - \frac{\tau}{\phi} - \frac{\rho}{\phi}\int_{\mathcal{S}} di$, with $\mathcal{I} = [0, i)$ one gets

$$\pi_i^\tau(\rho) = \pi - \frac{1}{2}\frac{\eta_i^2}{\phi} - \frac{1}{2}\frac{\tau^2}{\phi} - \frac{\tau(\bar{\eta} + \rho i)}{\phi}. \tag{69}$$

Substituting one has

$$\frac{d}{di}(\frac{\eta_i}{\rho} - 1)\pi_i^\tau(\rho) = \frac{\eta_i'}{\rho}\pi_i^\tau(\rho) - (\frac{\eta_i}{\rho} - 1)(\eta_i\frac{\eta_i'}{\phi} + \frac{\rho}{\phi}). \tag{70}$$

Therefore, recalling that $\eta_i' > 0$, this simplifies to

$$\pi_i^\tau(\rho) > (\eta_i - \rho)(\frac{\eta_i}{\phi} + \frac{\rho^2}{\phi\eta_i'}). \tag{71}$$

A sufficient condition is that the productivity of the dirtiest technology satisfies

$$\pi > \frac{3}{2}\frac{\eta_i^2}{\phi} + \frac{\eta_i\rho^2}{\phi\eta_i'} + \frac{1}{2}\frac{\tau^2}{\phi} + \frac{\tau(\bar{\eta} + \rho i)}{\phi}. \tag{72}$$

It is worth noting that this condition is in line with the preliminary assumption of large π so that it is always optimal to produce some level of emissions for each type i.

Proof. [**Proposition 7**] From the definition of equilibrium, we look for a cutoff type $x \in (0, 1)$ and a rate ρ such that the following conditions are jointly verified

$$\int_0^x \frac{\rho}{\phi}h_i di = \int_x^1 \frac{\pi_i^\tau(\rho)}{\rho}h_i di \quad \text{and} \quad \Pi_i^\tau(\rho) = 0 \quad \text{for} \quad i = x. \tag{73}$$

Recalling $h_i = \$1$ for each i and that $\pi_i^\tau(\rho)$ is as in (68), the market clearing condition implies that

$$\frac{\rho^2 x}{\phi} + \rho\frac{\tau x(1 - x)}{\phi} = (\pi - \frac{1}{2}\frac{\tau^2}{\phi} - \frac{\tau\bar{\eta}}{\phi})(1 - x) - \int_x^1 \frac{1}{2}\frac{\eta_i^2}{\phi}di, \tag{74}$$

which rearranging gives

$$\rho = \frac{-\tau(1 - x) + \sqrt{(\tau(1 - x))^2 + 4\phi k_x}}{2} \tag{75}$$

with $k_x = \frac{1}{x}(\pi - \frac{1}{2}\frac{\tau^2}{\phi} - \frac{\tau\bar{\eta}}{\phi})(1 - x) - \frac{1}{x}\int_x^1 \frac{1}{2}\frac{\eta_i^2}{\phi}di$. The indifference condition implies that

$$\frac{1}{2}\frac{\rho^2}{\phi} = (\eta_x - \rho)\frac{\pi_x^\tau(\rho)}{\rho}, \tag{76}$$

which rearranging gives the result. Now if $\rho + \eta_x > \phi - \tau$, then the equilibrium is non-consistent in that we have reached a corner solution in which the rate ρ is so high that the optimal abatement technologies of some issuers $0 < i < x$ go beyond the available cleanest technology $\delta = 1$. This is the case where the emissions reduction is priced at a rate that would encourage entrepreneurs to invest in technologies with negative emissions. In such a case, the equilibrium rate is set by the type whose abatement technology in

equilibrium $\delta_x = 1$, that is,

$$\rho = \eta_x = \frac{\phi - \tau}{2}. \tag{77}$$

In such a corner solution, there is a technology-constrained carbon-contingent financing in equilibrium with abatement supply $\frac{(\phi - \tau)}{2\phi} x$.

Voting problem. Let the median voter be a seller of carbon contingent securities. The utility for a given tax τ is given by

$$\mathcal{I}_i^\tau = U_i^\tau + \frac{1}{2} \frac{(\rho^\tau)^2}{\phi} \tag{78}$$

where U_i^τ is given by

$$
\begin{aligned}
U_i^\tau &= \pi - \phi(\delta_i^\tau) - \eta_i e_i^\tau - \tau e_i^\tau + \alpha \tau - \lambda E^\tau \\
&= \pi + \frac{1}{2} \frac{\eta_i^2}{\phi} - \frac{1}{2} \frac{\tau^2}{\phi} + \tau \frac{\eta_i - \bar\eta - \rho x}{\phi} - \lambda E^\tau.
\end{aligned} \tag{79}
$$

Denote U_i^* the utility in the laissez-faire with financial markets where

$$U_i^* = U_i^0 = \pi + \frac{1}{2} \frac{\eta_i^2}{\phi} - \lambda E^* \tag{80}$$

with $E^* = 1 - \frac{\bar\eta + \rho^* x^*}{\phi}$. Therefore $\mathcal{I}_i^\tau - \mathcal{I}_i^*$ gives

$$
\begin{aligned}
\mathcal{I}_i^\tau - \mathcal{I}_i^* &= U_i^\tau - U_i^* - \frac{1}{2} \frac{((\rho^*)^2 - (\rho)^2)}{\phi} \\
&= -\frac{1}{2} \frac{\tau^2}{\phi} + \tau \frac{\eta_i - \bar\eta - \rho x}{\phi} + \lambda(E^* - E^\tau) - \frac{1}{2} \frac{((\rho^*)^2 - (\rho)^2)}{\phi} \\
&= -\frac{1}{2} \frac{\tau^2}{\phi} + \tau \frac{\eta_i - \bar\eta - \rho x}{\phi} + \lambda(\frac{\rho x + \tau}{\phi} - \frac{\rho^* x^*}{\phi}) - \frac{1}{2} \frac{((\rho^*)^2 - (\rho)^2)}{\phi}
\end{aligned} \tag{81}
$$

which is zero in $\tau = 0$, concave in τ with a maximum at $\tau_i \approx \lambda + (\eta_i - \bar\eta) - \rho x$, and decreasing afterwards. Solving for τ such that $\mathcal{I}_i^\tau - \mathcal{I}_i^*$ gives the result.
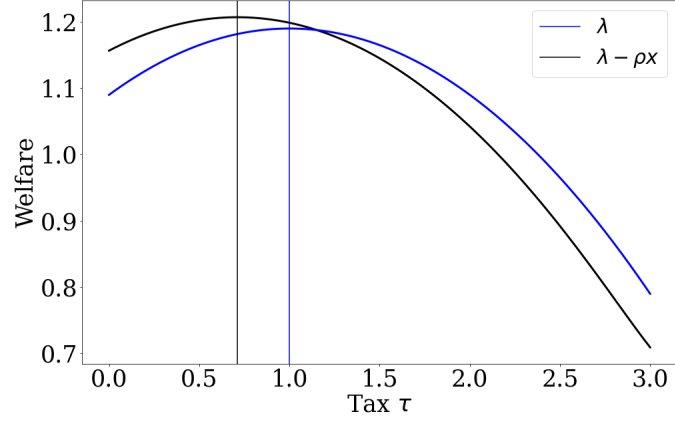
Proof. [**Proposition 8**] From the utility of the lender and issuer of carbon contingent securities, the utilitarian welfare reads

$$W^\tau(\rho) = \int_x^1 (\pi_i^\tau(\rho) - \rho q_i^\tau(\rho) - \eta_i(e_i^\tau - q_i^\tau(\rho))) di + \int_0^x (\pi_i^\tau(\rho) - \eta_i e_i^\tau(\delta_i^\tau(\rho)) + \rho(e_i^\tau - e_i^\tau(\delta_i^\tau(\rho)))) - \lambda E^\tau(\rho) \tag{82}$$

with $E^\tau(\rho) = E^* - \frac{\tau}{\phi} - \frac{\rho x}{\phi}$. From the market clearing condition and recalling the revenue-neutrality of

Figure 8. Welfare and Carbon Contingent Financing

The plot shows the utilitarian welfare in (82) (black line) as a function of the tax τ comparing it with the baseline welfare in absence of financial markets (blue line).

the carbon tax, this simplifies to

$$
\begin{aligned}
W^\tau(\rho) &= \int_x^1 (\pi - \frac{1}{2}\frac{(\eta_i + \tau)^2}{\phi} - \eta_i(e_i^\tau - q_i^\tau(\rho)))di + \int_0^x (\pi - \frac{1}{2}\frac{(\eta_i + \tau + \rho)^2}{\phi} - \eta_i e_i^\tau(\delta_i^\tau(\rho))) - \lambda E^\tau(\rho) \\
&= \int_0^1 (\pi + \frac{1}{2}\frac{\eta_i^2}{\phi} - \eta_i)di + \int_x^1 \eta_i q_i^\tau(\rho)di - \frac{1}{2}\frac{\tau^2}{\phi} - \frac{1}{2}\frac{\rho^2 x}{\phi} - \frac{\rho x \tau}{\phi} - \lambda E^\tau(\rho) \\
&= W^* + f(\rho) - \frac{1}{2}\frac{\tau^2}{\phi} - \frac{1}{2}\frac{\rho^2 x}{\phi} - \frac{\rho x \tau}{\phi} + \lambda(E^* - E^\tau(\rho)) \\
&= W^* + f(\rho) - \frac{1}{2}\frac{\tau^2}{\phi} - \frac{1}{2}\frac{\rho^2 x}{\phi} - \frac{\rho x \tau}{\phi} + \lambda(\frac{\tau}{\phi} + \frac{\rho x}{\phi}),
\end{aligned}
$$

(83)

which is a concave function of τ as shown in Figure 8. The solution τ^o therefore satisfies $\frac{d}{d\tau}W^\tau(\rho) = 0$, which gives the result. Note that (83) can be rewritten as

$$
W^\tau(\rho) = W^\tau + f(\rho) - \frac{1}{2}\frac{\rho^2 x}{\phi} - \frac{\rho x \tau}{\phi} + \lambda\frac{\rho x}{\phi}.
$$

(84)

Since the unconstrained solution $\tau^o \approx \lambda - \rho^o x^o$, the expression simplifies to

$$
W^{\tau^o}(\rho^o) = W^\tau + f(\rho^o) + \frac{1}{2}\frac{(\rho^o)^2 x^o}{\phi} \geq W^\tau,
$$

(85)

which proves the result since $f(\rho^o) \geq 0$.

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