Variable Information Quality, Ambiguity, and Stock Returns

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Abstract

This paper shows that higher quality of information can increase both conditional and unconditional equity premium in a model with ambiguity aversion to signals. I extract the time series of information quality from data on professional forecasters while differentiating information quality from uncertainty, volatility, and cross-sectional forecast dispersion. I find that, in the U.S. stock market, one standard deviation increase in information quality predicts a 3% increase in quarterly excess market returns with R^2 up to 7.4% (and out-of-sample R^2 up to 6.1%). Consistent with the implication of the model, information quality also increases the predictability of the dividend-price ratio. Three components of the model-implied stochastic discount factor—information quality, uncertainty, and aggregate signal—explain up to 83% of the cross-sectional variation in the average returns on size, book-to-market, and momentum-sorted portfolios.

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1 Introduction

Uncertainty and information are central tenets of finance. Investors can gather more information to attenuate uncertainty, or uncertainty can rise because of unexpected information. Such dynamic interaction between uncertainty and information can govern an investor's behavior and equilibrium asset prices. Information quality can be a fundamental determinant of asset prices, because it is a main environmental factor of the interaction between uncertainty and information. However, many unanswered questions remain regarding the size and direction of these effects on asset prices.

Uncertainty in the economy seems to be time-varying, but does information quality, as an environmental determinant, change over time? If so, do asset prices or conditional equity premia rise when we expect more precise information? How does more variability of information quality affect the conditional and unconditional equity premia? If investors have more doubts about tomorrow's information, does that make any difference in equilibrium asset prices? Are uncertainty and information quality contemporaneously correlated or does one lead the other? Do investors perceive lower information quality on good news than on bad news?

To answer the above questions, I start by building an asset pricing model that generates interesting asset pricing implications regarding variable information quality, following the approach proposed by Ju and Miao (2012). They assume that consumption growth follows a two-state Markov-switching process where a single representative agent learns about the state of the economy from consumption growth history and displays ambiguity aversion to the future state of the economy. To investigate the role of information quality in asset pricing, I modify three things from their model. First, the agent receives an extra signal from the *future* state of the economy. In other words, the learning process in my model is from *foresight* rather than *hindsight*. Second, I assume that the agent displays ambiguity aversion to future *information* rather than the future state of the economy. Finally, information quality is time-varying and independent of all other shocks in the economy.[1]

One might wonder why information quality is so important. Nowadays investors are living in an environment with a plethora of information. Their investment decisions and the resulting equilibrium price depend on what information they have and how they process it. Recent developments in information technology have exposed investors to much more information at lower cost. Thus, processing information becomes more important than merely

[1] In true dynamics or data, shocks can be correlated. Independence is assumed only because it makes the model generate the purely endogenous effects.

acquiring it. Consider a simple Bayesian belief-updating procedure:

$$u_t^* = (u_t^{-1} + q_t)^{-1} \quad \text{and} \quad \hat{\mu}_t^* = [1 - u_t^* q_t]\,\hat{\mu}_t + [u_t^* q_t]\,y_t \tag{1.1}$$

In this simple Bayesian belief-updating procedure, information quality is the main determinant of information processing. Information quality (as precision q_t) plays two roles: it determines how much uncertainty can be reduced from u_t to u_t^*, and it also changes the investor's beliefs from $\hat{\mu}_t$ to $\hat{\mu}_t^*$ through the signal y_t. Another interesting question arises here. Why does an investor display ambiguity aversion to the future signals in the first place?[2] When there is high uncertainty in the economy but she can collect or receive high quality information, the potential influence of information is high as well. Thus, a slight miscalculation of information quality has large negative consequences. In this scenario, it is natural for her to display ambiguity aversion to future signals, and the size of ambiguity increases with information quality and uncertainty.[3]

Another plausible story is related to rational inattention or bounded rationality. An investor knows she will receive a number of signals in the sense that each bit of news and each personal observation is an individual signal. She also knows that there will be much information to which she will not pay attention or have access. It is beyond her intellectual ability to assign a proper prior distribution to each of these future signals. Thus, she instead decides to aggregate all the signals into one hypothetical signal that approximates the signal in the simple Bayesian belief-updating process shown in Equation (1.1). However, the aggregation procedure is so doubtful that she can not impose a proper prior distribution on the future aggregate signal. This explanation also provides justification for ambiguity aversion to future signals.

In many cases, different models provide opposing implications. This is actually the case in this paper. First, the model I develop here predicts that higher information quality today forecasts higher excess market return tomorrow, whereas pre-existing models predict the opposite. Second, my model predicts that information quality will improve return predictability of the dividend price ratio, while others do not. This is good news to an empiricist: difference in model implications offers a great opportunity to evaluate the models using actual data. The only issue is whether proper data and econometric methods are available.

Unfortunately, information quality as perceived by an investor is unobservable. To make matters worse, information quality is closely related to uncertainty, volatility, and heterogene-

[2] Ambiguity aversion to signals in this paper is different from a model with ambiguous signals proposed by Epstein and Schneider (2008).

[3] The difference between ambiguity and ambiguity aversion is explained in Section 2.

ity.[4] When an empiricist uses one of these variables as a proxy for information quality, it is not possible to differentiate the proxy from information quality itself. Therefore, an empirical result that uses such a proxy may not be an accurate measure of the effect of information quality.[5]

To tackle this empirical issue, I take a different approach to utilizing data. Instead of selecting a directly observable proxy, I construct an econometric model that is robust to model misspecification and consistent with the probabilistic environment of the economic model I develop.[6] There is a trade-off in choosing a probabilistic environment for the economic model over the purely data-oriented one for the econometric model. The economic model consists of a probabilistic environment and a decision maker's preference. Thus, an empirical result can be misleading if a probabilistic environment in the data is too different from the one in the economic model and an empiricist directly fits the economic model with data. Likewise, an empirical result can fail to fairly compare different economic models if an empiricist uses a fully data-oriented probabilistic environment that is too different from the one in the model.[7]

Considering this trade-off, I extract time-varying information quality, uncertainty, and aggregate news from the survey data *Survey of Professional Forecasters*. To implement this empirical part of the study, I design the extraction procedure to be a statistical filtering problem that can be easily solved by a nonlinear filter. After obtaining the time-series of information quality, uncertainty, and aggregate news, I use them to perform a standard return prediction analysis and a cross-sectional analysis. I find that variable information quality, as predicted by the model, strongly forecasts the future excess market returns. In the U.S. stock market, one standard deviation increase in information quality predicts a 2.0% to 3.5% increase in quarterly excess market returns with R^2 of 4.6% to 7.4% (and an out-of-sample R^2 up to 6.1%). Interestingly, information quality controlled by uncertainty shows better return-predictability especially in the longer horizon. Furthermore, information quality dramatically enhances the return-predictability of the dividend-price ratio. At two-year horizon, R^2 of predictive regression jumps from 6.8% to 37.6% when information quality is included as an additional predictor. All these results are consistent with what my model predicts. On the other hand, three components of the model-implied stochastic discount factor—information

[4] In a Kalman filtering scheme, uncertainty is proportional to a reciprocal of signal precision. Even though information quality changes over time, uncertainty quickly converges to be proportional to information quality. Thus, they move together as assumed in Bansal and Shaliastovich (2009). On the other hand, if an agent learns about the state of the economy only from the history of past consumption growth, consumption volatility is a reciprocal of signal precision. Heterogeneity is often empirically interpreted as market-wide uncertainty.

[5] A popular choice for the proxy is cross-sectional forecast dispersion.

[6] In the model proposed in this paper, a probabilistic environment is dynamics of the economy and structure of signals.

[7] However, the latter correctly provides an empirical result regardless of its interpretation in connection with a pricing model.

quality, prior uncertainty, and aggregate signal—account for the cross-sectional variation in expected returns by size, book-to-market, and momentum-sorted portfolios. By-products of my empirical procedure provide evidence that (1) investors do not consider bad news to be more reliable than good news, contrary to the assumption in Epstein and Schneider (2008) and (2) prior uncertainty and information quality are positively correlated, implying a stronger return predictability of information quality than would be implied if they were independent.[8]

In sum, the contribution of this paper is threefold. First, I construct an asset pricing model with ambiguity aversion to signal, which generates interesting asset pricing and econometric implications. Second, I develop a new approach to applying a nonlinear filter to the survey data. The approach is robust to misspecification of parameters and models. Finally, I extract information quality from the survey data and find striking empirical results that are consistent with model predictions. These three contributions are intertwined and complete one another throughout this paper.

The remainder of this paper is organized as follows. Section 1.1 reviews existing literature related to this paper. Section 2 describes an economy with an ambiguous signal and time-varying signal quality. Sections 3 induces asset-pricing and econometric implications regarding return predictability. It compares different types of preferences and information quality dynamics. Section 4 describes the data and the structural model designed to extract variable information quality from the data on professional forecasters. Section 5 describes the econometric methods used to solve a filtering problem defined in Section 4. Section 6 reports the empirical results on filtered variables, return predictability, and cross-sectional test. Section 7 discusses outside validation and suggests how to improve return prediction performance for practical purposes. Section 8 offers some concluding remarks and proposes future research ideas.

1.1 Literature Review

Ambiguity. Since the Ellsberg Paradox brought attention to ambiguity (or Knightian uncertainty), several different approaches have been developed to quantify the effect of ambiguity.[9] Epstein and Schneider (2003) devise a multiple prior approach to design a decision maker's reluctance to impose one fixed prior distribution on the unknown parameters. In the same max-min framework, Epstein and Schneider (2007) model learning under ambiguity. On the other hand, Klibanoff et al. (2009) develop a smooth ambiguity approach, and

[8]Positive correlation between prior uncertainty and information quality can be interpreted as an investor's endogenous information-acquisition behavior.

[9]See Ellsberg (1961).

Klibanoff et al. (2005) extend it to a recursive setting. Hansen and Sargent (2007) formulate a learning model differentiating concern about the dynamics of hidden states and concern about current value of hidden states. Hansen (2007) shows how equilibrium prices reflect an investor's concern about statistical ambiguity. Hansen and Sargent (2011) use statistical detection theory in a continuous-time environment to calibrate ambiguity aversion. Ju and Miao (2012) develop a generalized version of recursive smooth ambiguity preference, preserving its homothetic form. Drechsler (2008) hardwires the degree of ambiguity into the variance of hidden macro state variables, then he restores the implied ambiguity from option prices. Anderson et al. (2009) provide an empirical study considering ambiguity and risk as two main components in asset pricing. Ulrich (2010) and Collard et al. (2011) incorporate the effect of learning and ambiguity within a long-run risk model framework.[10]

Ambiguous Signal. It is well known that information can be ambiguous to investors. Leippold et al. (2008) explain the size of equity premium and volatility by introducing ambiguous information and investors' learning. Caskey (2009) utilizes a smooth ambiguity approach. Both papers assume ambiguity in the mean of signal. On the other hand, Epstein and Schneider (2008) model investors' concern about signal precision with a multiple-prior approach. In their model, asset valuation is affected even before an ambiguous signal arrives only if an investor conceives she will get an ambiguous signal.[11]

Information Quality. Veronesi (2000) shows that a precise signal can increase the unconditional equity premium in an endowment economy populated by investors with CRRA preference. Ai (2010) argues that a precise signal decreases equity premium in a production economy with long-run risk. Brevik and d'Addona (2010) also find that with a recursive utility, higher information quality decreases equity premium when the preference parameters are calibrated by Bansal and Yaron (2004). Li (2005) also finds that a precise signal decreases equity premium. Vanden (2008) studies the relation between information quality and option prices. Gollier and Schlee (2011) use a different signal structure and show that information about volatility can raise the equity premium for a wide class of preferences. However, all these papers study the effect of information quality as comparative statistics. Bansal and Shaliastovich (2009) build a setting with time-varying information quality, but information quality and uncertainty are not separated in their model. On the other hand, uncertainty has been considered a novel important state variable in a pricing kernel. Ozoguz (2009) shows

[10] There are many papers that study the effect of learning within a long-run risk model framework. See Pakos and Chen (2008), Shaliastovich (2008), Bansal et al. (2010), Croce et al. (2007), and Johannes et al. (2010).

[11] The model in this paper also shows that asset prices are affected by ambiguity aversion in an investor's preference even if signal is uninformative.

empirically that an investor's uncertainty explains the cross-sectional variation in expected returns.

Survey Data and Heterogeneous belief. Existing literature in macroeconomics and econometrics focuses on evaluating the forecasting ability or the rationality of the forecasters. Some papers address the source of dispersion of forecasts and the relation between uncertainty and heterogeneity. Dopke and Fritsche (2006) study inflation forecast dispersion. Patton and Timmermann (2008) and Patton and Timmermann (2010) focus on the term structure of cross-sectional dispersion. Lahiri and Sheng (2010) provide an empirical measure of uncertainty using disagreement in survey data. David and Veronesi (2008) take a structural form approach to forecast volatility considering inflation and earnings uncertainty. Survey data inevitably show heterogeneity in investors' beliefs. When survey data is used for a single representative model, heterogeneity plays the role of market-wide uncertainty. From cross-sectional dispersion of analysts' earnings forecasts, Buraschi et al. (2009) explain the relation between correlation premium and option prices. Diether et al. (2002) find empirical evidence that cross-sectional dispersion among analysts explains the cross-sectional variation in the expected returns.[12] Scheinkman and Xiong (2003) and David (2008) provide a theoretical framework for a role of heterogeneity in financial market.

Return Predictability. Cochrane (2008) and Cochrane (2011) emphasize the importance of the dividend price ratio in return prediction. Pastor and Stambaugh (2009) and Van Binsbergen and Koijen (2010) introduce a hidden state variables to construct a better predictor. Goyal and Welch (2008) compare the performance of competing predictors in various settings. Johannes et al. (2009a) utilize a sequential parameter learning algorithm to improve out-of-sample predictability. Kelly (2011) extracts tail risk premia from cross-sectional stock returns and shows that time-varying tail risk can predict future excess market returns. Kelly and Pruitt (2011a) maximize prediction performance of dividend price ratio with a filtering method developed in Kelly and Pruitt (2011b).

2 Ambiguity Aversion and Variable Information Quality

2.1 Generalized Recursive Smooth Ambiguity Utility

Ju and Miao (2012) propose a generalized recursive ambiguity utility model that permits a three-way separation among risk aversion, ambiguity aversion, and intertemporal substitu-

[12]Their result is consistent with the return predictability of information quality in this paper. Johnson (2004) develops a theory explaining their result.

tion. It has the same recursive utility form developed by Epstein and Zin (1989):

$$V_t(C) = \left[(1-\beta)C_t^{1-\rho} + \beta\left\{\mathcal{R}_t(V_{t+1}(C))\right\}^{1-\rho}\right]^{\frac{1}{1-\rho}} \tag{2.1}$$

where ρ^{-1} is the elasticity of intertemporal substitution (EIS), β is the subjective discount factor, γ is the risk aversion parameter, and η is the ambiguity aversion parameter. However, Ju and Miao (2012) assume the uncertainty aggregator $\mathcal{R}_t$ has a more generalized homothetic form with returns given by:

$$\mathcal{R}_t(V_{t+1}(C)) = \left\{\mathbb{E}_{\pi_t}\left(\mathbb{E}_{\pi_t(z_{t+1})}\left[V_{t+1}^{1-\gamma}(C)\right]\right)^{\frac{1-\eta}{1-\gamma}}\right\}^{\frac{1}{1-\eta}} \tag{2.2}$$

where π_t is the posterior distribution of the representative agent formed by information available up to time t, and $\pi_t(z_{t+1})$ is the predictive distribution conditioned on the state variable z_{t+1}. Here, it is generally assumed that the agent is updating her belief by Bayes' theorem and that $\{z_{t+1}\}$ is unobservable at time t. Ju and Miao (2012) assert that: (i) the inside expectation captures the aversion to the risk of transitory shocks in consumption growth, and (ii) by Jensen's inequality, the outside expectation characterizes ambiguity aversion to the specific parameters or state variables when the ambiguity aversion parameter η is larger than the risk aversion parameter γ.

Ju and Miao (2012) consider the convenience of comparative statics analysis and analytic tractability to be the main advantages of this utility. They further emphasize that the smooth ambiguity preference separates ambiguity attitudes from ambiguity itself.[13]

However, there is an additional benefit in using this form of preference. Because of the flexibility of this utility form, uncertainty in each state variable (or parameter) can be assigned a different degree of aversion. Simply put, we can choose any state variable to be $\{z_{t+1}\}$ in Equation (2.2). Ju and Miao (2012) focus on the setting in which the agent dislikes uncertainty in persistent shocks more than uncertainty in transitory shocks. I put my focus on uncertainty in the quality of information.

2.2 Economic Model

Dynamics of the Economy. Considering the effect of learning and the quality of information, I set up a representative agent model in a pure-exchange economy. I assume that there is only one consumption good, and at each period aggregate consumption $\{C_t\}$ is

[13]Ambiguity is characterized by the subjective set of measures or distortion in belief held by an agent who prefers a robust decision. Ambiguity attitudes are characterized by the size of η relative to γ. See Ghirardato and Marinacci (2002), Klibanoff et al. (2005), and Epstein (1999) for the foundation of ambiguity attitudes and ambiguity.

endowed to the whole economy. Agents trade multiple risky assets with bonds that have a one-period maturity and net supply is zero at equilibrium. The market portfolio of equities pays the aggregate dividends $\{D_t\}$. Following most of the learning-based models in recent literature, aggregate consumption growth follows a two-state Markov-switching process:[14]

$$\log \frac{C_{t+1}}{C_t} = \mu_t + \sigma_c \epsilon_{c,t+1} \tag{2.3}$$

where μ_t switches between the low mean growth state μ_L (recession) and the high mean growth state μ_H (boom) which is unobservable to the agents.[15] The transition probability matrix $\mathbb{P}_\mu$ is

$$Pr(\mu_{t+1}|\mu_t) = \mathbb{P}_\mu = \begin{bmatrix} p_{ll} & 1-p_{ll} \\ 1-p_{hh} & p_{hh} \end{bmatrix} \tag{2.4}$$

As noted by Ju and Miao (2012), the exogenous fluctuation in the conditional consumption volatility is not necessary to generate time-variation in risk premium in the model with agents' learning. However, agents' learning is not the the main source of time-varying risk premium either. Even without the effect of learning, the equity premium and the dividend-price ratio, though they take only discrete values, can fluctuate in a Markov-switching economy.

As specified in Johannes et al. (2010), Boguth and Kuehn (2009), and Bansal and Yaron (2004), the dividend process follows

$$\log \frac{D_{t+1}}{D_t} = \phi_s \mu_t + g_d + \phi_c \sigma_c \epsilon_{c,t+1} + \sigma_d \epsilon_{d,t+1} \tag{2.5}$$

Temporary shocks $\epsilon_{c,t+1}$ and $\epsilon_{d,t+1}$ are identically and independently distributed standard Normal random variables, and they are independent of the regime-switching shocks. This dividend specification as levered consumption effectively connects consumption and dividend without introducing extra state variables. I generalize the dividend process by separating ϕ_s from ϕ_c in Equation (2.5). When μ_t is not observable to the agent, the agent also learns from the dividend if $\phi_s \neq \phi_c$.

Information Structure. So far, the model in this paper has been identical to the model studied in Ju and Miao (2012). However, I propose two important changes in the model specification. First, I assume that the agent observes, in addition to a history of consumption

[14]See Ju and Miao (2012), Johannes et al. (2010), Boguth and Kuehn (2009), Cecchetti et al. (1990, 1993, 2000), and Kandel and Stambaugh (1991).

[15]Learning-based Models with uncertainty in the mean growth state are studied in Pakos and Chen (2008), Bansal and Shaliastovich (2009), Shaliastovich (2008), Ulrich (2010), Hansen et al. (2008), Bansal et al. (2010), and Croce et al. (2007).

and dividend, a signal $\{y_t\}$ about the true state of the economy μ_t whose value can take either μ_L (recession) or μ_H (boom). Here, the source and timing are crucial in drawing implications from the model. Existing models with additional signals assume that the signal comes from the state as the conditional mean of the current consumption growth $\log \frac{C_t}{C_{t-1}}$. For example, many models assume $y_t \sim \mathcal{N}(\mu_{t-1}, \sigma_c^2)$ and $\log \frac{C_t}{C_{t-1}} \sim \mathcal{N}(\mu_{t-1}, \sigma_c^2)$. However, in this paper, the agent receives the signal y_t at time t, and y_t is related to μ_t, the conditional mean of the next period's consumption growth. I will discuss more about the signal timing in the next subsection.

I specify a discrete signal structure to simplify the analysis. Signal $\{y_t\}$ is a identically and independently distributed binomial random variable whose value takes either μ_L or μ_H. As illustrated below, the probability that the signal $\{y_t\}$ correctly indicates the true mean growth state is $q_t \geq 0.5$ which is observable at the time of arrival of signal $\{y_t\}$:

Signal	True state	
y_t	μ_H	μ_L
μ_H	q_t	$1 - q_t$
μ_L	$1 - q_t$	q_t

I interpret $\{q_t\}$ as the time-varying quality of information, and $\{q_t\}$ also follows a two-state Markov-switching process with a transition probability matrix $\mathbb{P}_q$ which is independent of all other random variables in the economy up to time t.[16] q_t can take a value of either q_A or q_I where $q_A \geq q_I \geq 0.5$. A transition probability matrix $\mathbb{P}_q$ is defined as

$$Pr(q_{t+1}|q_t) = \mathbb{P}_q = \begin{bmatrix} p_{ii} & 1 - p_{ii} \\ 1 - p_{aa} & p_{aa} \end{bmatrix} \tag{2.6}$$

where $p_{aa} = Pr(q_{t+1} = q_A | q_t = q_A)$ and $p_{ii} = Pr(q_{t+1} = q_I | q_t = q_I)$. The other departure from a group of existing ambiguity models is a choice of a variable to which ambiguity aversion is attached. I also define Prior Uncertainty U_t and Posterior Uncertainty U_t^* as

$$U_t = var_{t-1}[\mu_t] \quad \text{and} \quad U_t^* = var_t[\mu_t] \tag{2.7}$$

Thus, Prior Uncertainty U_t is the variance of μ_t in an agent's belief before she receives a signal y_t, while Posterior Uncertainty U_t^* is the variance of μ_t in an agent's belief after she receives a signal y.

[16]When $\{q_t\}$ is correlated with uncertainty of the economy or the mean growth state itself, the size and the sign of correlation affect the equity premium. See Yae (2011b) for the detailed analysis.

Ambiguity Aversion to Signals. Another important modification is a different choice of state variable to which an agent shows ambiguity aversion. Unlike Ju and Miao (2012) or Collard et al. (2011), I assume the agent displays ambiguity aversion to the signal she receives at the next period. That is, $z_{t+1} = q_{t+1}$. This is more than a new choice of ambiguous variable. With this specification of ambiguity aversion, the agent does not directly distort her belief on dynamics of the economy, either in states or in parameters.[17]

I also set the dynamics of the quality of information to follow a two-state Markov-switching process independent of any other shock in the economy up to time t. The probability of staying in the current state is set as 90% regardless of the current state of information quality in Table 2.

An interesting implication of this specification is that ambiguity, not ambiguity aversion η, becomes larger when agents perceive that there will be higher quality of information tomorrow.[18] Since the higher quality of information will force the agents' beliefs in the future to be more volatile, the agents' value function drops due to ambiguity aversion. The effect of time-varying ambiguity is also studied in Drechsler (2008) with a recursive multiple prior approach. In his model, ambiguity is proportional to the variance of the shocks in the persistent component of consumption growth, which corresponds to the Markov-switching jumps of the model in this paper.

In the current framework, as long as the agent's value function does not depend on the efforts of acquiring information and the dynamics of information quality remain unchanged, the source of time variation in information quality does not matter. In other words, implications from the model are blind to whether there is an endogenous acquisition of information or an exogenous shock in the quality of information. Due to this observational equivalence, time-varying information quality can be interpreted as a change in an investor's sentiment and behavioral attitude, as time-varying bounded rationality, or as time-varying capacity of rational inattention.

2.3 Asset Pricing

Following Duffie and Skiadas (1994) and Hansen et al. (2008), Ju and Miao (2012) derive the stochastic discount factor of the generalized recursive smooth ambiguity model. They utilize the homogeneity property of the value function in Equation (2.1). The stochastic discount

[17]This will be discussed later with the return-predictability of the dividend-price ratio.

[18]Equivalently, ambiguity becomes larger when the agent perceives the high quality of information today, and the quality of information is persistently time-varying, i.e., $p_{aa} > 0.5$ and $p_{ii} > 0.5$.

factor has the form

$$M_{z_{t+1},t+1} = \beta \left(\frac{C_{t+1}}{C_t} \right)^{-\gamma} \left(\frac{C_t}{C_{t+1}} \frac{V_{t+1}}{\mathcal{R}_t(v_{t+1})} \right)^{\rho-\gamma} \left(\frac{\mathbb{E}_{\pi_t(z_{t+1})}[V_{t+1}^{1-\gamma}]^{\frac{1}{1-\gamma}}}{\mathcal{R}_t(v_{t+1})} \right)^{\gamma-\eta} \tag{2.8}$$

where $\mathbb{E}_{\pi_t(z_{t+1})}$ denotes the expectation operator under the measure $\pi_t(z_{t+1})$, which is the conditional expectation given the information up to time t plus hypothetical knowledge of the state z_{t+1}. The first and second components of this stochastic discount factor are short- and long-run consumption risk, respectively. The last component comes from ambiguity aversion.[19] More detailed interpretation and analysis of this stochastic discount factor are given in Ju and Miao (2012).

The Euler equation with this stochastic discount factor provides the solution:

$$1 = \mathbb{E}_{\pi_t} \left[M_{z_{t+1},t+1} R_{i,t+1} \right] \tag{2.9}$$

where $R_{i,t+1}$ is a gross return of any asset i, and $\mathbb{E}_{\pi_t}$ denotes the expectation under the measure π_t, a conditional expectation given the information up to time t. The details of finding the asset pricing solution are described in the Technical Appendix in Ju and Miao (2012).

3 Asset Pricing Implications of Model

3.1 Model Characteristics

Foresight, Hindsight, and Learning. In the model in Section 2, an agent receives a signal y_t for the mean growth of the next period μ_t, instead of the current period μ_{t-1}.[20] In other words, the model is designed to examine the effect of foresight while the existing literature focuses on hindsight. Learning from foresight can be very different from hindsight. In the case of hindsight, even if an agent receives perfect information about the current state of the economy, the economy in the future can be still uncertain to that agent.[21] For example, the calibrated model in Ju and Miao (2012) implies that an agent puts a chance of another recession (low mean growth state) next year at about 50 percent, even though she knows

[19] If $\eta = \gamma$, the stochastic discount factor reduces to that of a standard recursive utility in Epstein and Zin (1989). If $\eta < \gamma$, the implication is that the agent is less averse to uncertainty in the specific state variable than to uncertainty in others.

[20] Equation (2.3) shows that the consumption growth at the current period is a normal random variable with the mean of μ_{t-1}.

[21] This is true especially when an agent displays an ambiguity aversion to the conditional mean of consumption or output growth.

exactly that the state μ_{t-1} is the low-mean growth state.[22] Obviously, this is why we call it hindsight: it does not help us make decisions about the future. On the other hand, an agent can be more certain about the future despite the same quality of signal when she learns from foresight. There is no predetermined limit on the degree of certainty in her belief. Therefore, a slight modification of the signal structure in the model can cause dramatic changes in asset pricing implications. Yae (2011b) provides a more detailed analysis on learning from hindsight and foresight.

Turning off Learning from Hindsight. Instead of solving a model with a hidden state variable, I simplify the model by assuming that at time t an agent can observe the state variable μ_{t-1} in addition to C_t, so she has perfect hindsight. This is in fact a simple modification of the limiting case of Brevik and d'Addona (2010) in which an agent receives signals with infinite precision for the state μ_{t-1}. The only difference is that the agent receives extra signals for μ_t and ambiguity aversion arises with respect to those signals. This simplified version of the model provides two important benefits. First, the asset pricing solution can be obtained simply by solving a couple of nonlinear equations. This enables us to investigate a number of interesting properties of the model with little time and effort. Thus, faster computation is still valuable, even though it is computationally tractable to solve the general model by numerical integration and a grid. More importantly, this simplification allows us to turn off the channel *learning from hindsight* and to focus on the effect of *learning from foresight* and ambiguity aversion. The only cost of this approach is that all the moments of the asset returns and the dividend-price ratio can take only a finite number of values since the state variables in the agent's continuation value are discrete. Smoothness in the solutions, however, is unnecessary for the purpose of this paper, since the simplified version of the model still bears the important asset pricing implications that are empirically tested here.[23]

Model Parameters. I use two different parameter sets for the Markov-switching dynamics of the economy. The first parameter set is the calibration result shown in Ju and Miao (2012) and Cecchetti et al.(2000). These parameter values are estimated with the data including the period of the Great Depression. The key characteristic of this parameter set is that the low-mean growth state, i.e., the bad state of the economy, is much lower than the one estimated with post-war data. The mean growth rate of -6.8% per year is so low that it can be interpreted as a disaster state rather than an ordinary recession. For this reason, I

[22]Persistence at the low-mean growth state is usually weaker because it implies a shorter average duration of recessions, as observed in the history, than that of boom periods. Learning from foresight can capture a time-varying transition probability matrix as emphasized in Ozoguz (2009), because it is possible to construct a hypothetical signal that gives an observational equivalence to a model with time-varying transition probability.

[23]See Yae (2011b) for more details about a general model with hidden states.

use a different parameter set in my other analysis. This second set of parameters has a less persistent transition matrix, a more moderate recession state, and a smaller consumption (or output) growth volatility, which are more consistent with post-war data.[24] The implied correlation between consumption growth and dividend growth is 0.76 in the parameter set of Ju and Miao (2012). I adjust the leverage of the dividend, ϕ_s and ϕ_c, in order to match the historical correlation.[25] I choose the other parameters for information quality to be persistent for the benchmark case in Table 1 and Figures 1, 2, 8, and 9. Average, time-variability, and persistence of information quality vary in Figures 3, 4, and 5, respectively. The preference parameters γ, η, and ρ^{-1} are specified in each column of Tables 1 and 9. For the other tables and figures, I used $\gamma = 2$, $\eta = 8.9$, and $\rho^{-1} = 1.5$ as benchmark values.[26] The values of both parameter sets are reported in Table 2.[27]

3.2 Size of Equity Premium

Panel A in Table 1 shows the different model implications between two ambiguity-based models: the model with ambiguity aversion to the mean growth states as in Ju and Miao (2012) and the model with ambiguity aversion to the signals. For each model, I compare the moments of excess returns according to three different settings of information. The first information setting is that the signals are just pure noise, characterized by 50% chance of the signal indicating the true state correctly, i.e., $q_t = q = 50\%$. In the second setting, the agent receives signals with higher quality, 75% chance of a correct indication. The last setting is the main focus of the paper. The quality of information is time-varying, although the average chance of a correct indication remains the same as in the second setting. When the quality of information is high, the signal correctly indicates the true state with 95% probability ($q_t = 95\%$). When the quality of information is low, $q_t = 55\%$.

Equity Premium Decomposition. Panel B in Table 1 and Figure 2 shows the decomposition of the equity premium. Several interesting results are observed with ambiguity aversion to signals. First, ambiguity aversion itself can generate an extra equity premium even when the signals are uninformative (i.e. pure noise). Here, it is 1.5% (annual) with

[24]Consumption growth and output growth are the same in the endowment economy I set up.

[25]I price a levered claim to consumption stream with a leverage factor $\phi_s = 4.5$ as in Bansal and Yaron(2004), Lettau, Ludvigson, and Watcher (2008), and Johannes et al. (2010). However, I set the temporary shock exposure $\phi_c = 1.5$ so that the correlation between consumption growth and dividend growth match the data.

[26]Ju and Miao (2012) use risk aversion parameter $\gamma = 2$ and ambiguity aversion parameter $\eta = 8.9$. They justify the size of ambiguity aversion with detailed discussion. They report that these values imply less than one tenth of ambiguity premium measured by Ellsberg-Paradox type experiments in Camerer(1999). The value of EIS $\rho^{-1} = 1.5$ is used in Bansal and Yaron(2004), Lettau, Ludvigson, and Watcher (2008), Ju and Miao (2012), and Johannes et al. (2010).

[27]The model implication I test in the paper is not sensitive to parameter values in general.

the parameter set II in Table 2. This result is consistent with Epstein and Schneider (2008) although the details in mechanism are different.[28] Second, overall information quality that is constant over time increases the unconditional equity premium by 1.7% (annual). This is a unique feature of the preferences I examine such as power utility, Epstein-Zin's recursive utility, and Ju-Miao's ambiguity aversion to the mean growth.[29] Figure 3 also shows the same phenomenon by a comparative statistics analysis approach. The intuition behind this result is simple. If the agent believes today that tomorrow there will be a high chance of receiving a very precise signal about the economy the day after tomorrow, then her value function will be either very high or very low tomorrow. She dislikes this situation and will dislike it much more if she has an ambiguity aversion to signals. Thus, her value function today becomes low. Therefore, her marginal utility and the equity premium rise. Third, time-variability of information quality increases the unconditional equity premium, while the average information quality is fixed at a constant as shown in Figure 4. That is, variable information quality, as a state variable, induces an extra equity premium from its hedging component in the stochastic discount factor. Finally, Figure 5 and Panel B in Table 1 show that the persistence of information quality can increase the unconditional equity premium. Long-run risk component plays a role in this.

3.3 Time-variation of Equity Premium.

Since Veronesi (2000), the effect of information quality has been studied in various economic settings and signal structures.[30] However, the existing literature has overlooked two important points. First, as discussed in the previous section, the role of foresight has not been incorporated into the models, partially because its implication is the same as that of hindsight if a commonly used preference is assumed in a model. Furthermore, building and solving a model with foresight is not straightforward in a continuous-time setting.[31] Second, the effect of information quality on equilibrium prices has been tackled mainly by comparative statistics analysis.[32] This is also difficult, since separating information quality dynamics from that

[28] In Epstein and Schneider (2008), the extra equity premium comes from ambiguous information quality, because an agent perceives information quality differently depending on whether it is good or bad news. In the model of this paper, the agent judges information quality independently of signal contents.

[29] Veronesi (2000) shows the same property with power utility. However, the result falls into the case of *learning from hindsight*. In my setting of *learning from foresight*, higher information quality decreases the equity premium even with power utility. Gollier and Schlee (2011) shows that information purely about the volatility either of consumption or the marginal utility of consumption raises the equity premium for a wide class of preferences.

[30] See Epstein and Schneider (2008), Ai (2010), Li (2005) Vanden (2008), and Gollier and Schlee (2011).

[31] For examples of models with hindsight in a continuous-time setting, see Veronesi (2000)

[32] Some papers consider the dynamics of information quality. Jacoby et al. (2011) takes ICAPM framework. Opp (2008) constructs a model with endogenous information acquisition. Bansal and Shaliastovich (2009) do not separate information quality dynamics from that of uncertainty.

of uncertainty is not straightforward. Presumably for these reasons, the premium from the hedging component has been ignored, and return-predictability of variable information quality has not been studied. In the model of this paper, unlike previous approaches, information quality can change over time completely independently of the states of the economy and prior uncertainty.[33]

Figure 4 shows that a larger time-variation of information quality can amplify time-variation of the conditional equity premium. Thus, time-variation of information quality can be a source of excess volatility. Figure 5 displays the effect of persistence in information quality dynamics. With sufficiently strong persistence, the conditional equity premium is higher when information quality is higher. The intuition behind this is similar to the story mentioned previously. If the persistence of information quality is high, then the agent is more certain about the information quality tomorrow. If today's information quality is low, then the agent believes tomorrow's information quality is also likely to be low. Her continuation value drops while the conditional equity premium increases. If persistence of information quality is not sufficiently strong, the relation between information quality and the conditional equity premium can be negative.

3.4 Return Predictability and Dividend-Price ratio.

My model generates interesting pricing implications which can be opportunities to empirically evaluate different learning-based models. Figures 6 and 7 display why information quality can better predict excess return when it is controlled by prior uncertainty or posterior uncertainty.[34] Figure 8 shows that information quality, as an additional predictor, can enhance the return-predictability of the dividend-price ratio. In my model, information quality plays a role as a control variable from the viewpoint of the regression analysis on excess market returns on a lagged dividend-price ratio. However, the other variables, such as uncertainty, do not play as significant a role as information quality does. In the other preference settings, the effect of information quality becomes much weaker as shown in Figure 9.

There is an intuition behind the role of information quality as a control variable. Unlike Ju and Miao (2012) or Collard et al. (2011), I assume that the agent displays ambiguity aversion to the signals she receives at the next period. This change has meaning beyond the new choice of *ambiguous* state variable. With this specification of ambiguity aversion, the distortion of agent's belief can be more independent of the dynamics of the economic fundamentals.[35]

[33]The term *prior uncertainty* denotes the uncertainty right before an agent receives a signal.

[34]The term *prior uncertainty* and *posterior uncertainty* denote the uncertainty before and after an agent receives a signal, respectively.

[35]In this statement, I categorize information quality, following a conventional thought, as an environment of learning rather than one of the economic fundamentals such as μ_t. However, the paper conclude information

Therefore, the effect of information quality on the conditional equity premium can be more independent of that of the dividend-price ratio. This is the intuition behind the role of information quality as a control variable.

4 Disentangling Information Quality from Uncertainty

4.1 Data

Macroeconomic and Financial Data. The real GDP growth data is taken from the Bureau of Economic Analysis.[36] For the test of return predictability, I proxy for the market portfolio using the return on the value-weighted CRSP index (NYSE/AMEX/NASDAQ). The excess market returns are obtained by subtracting CRSP 90-day T-bill rates from the market portfolio returns. The existing return predictors are taken from Amit Goyal's website.[37] For the cross-sectional analysis, I take Size-sorted, Book-to-market-sorted, and Momentum-sorted portfolios from Kenneth French's website, as well as the Fama-French three factors and the Momentum factor.[38]

Survey Data. Investors' beliefs about the macro economy are important to financial economists since they affect asset prices in the financial market. The Federal Reserve Bank of Philadelphia's *Survey of Professional Forecasters* (henceforth SPF) provides quarterly-updated forecasts by a group of professionals on many important macro variables, including real GDP growth, inflation, corporate profits, and unemployment.[39] While macro-economists focus on measuring forecasting performance or testing the rationality of forecasters, financial economists try to find a proxy for the level of heterogeneity and uncertainty from the disagreement amongst forecasters.[40] The most popular choice for the proxy is the cross-sectional forecast dispersion. This dispersion variable has been repeatedly used as a proxy for slightly different variables in several models.[41]

quality can be more important than the economic fundamentals because an agent can only observe signals which is totally characterized by information quality.

[36]http://www.bea.gov/national/nipaweb/SelectTable.asp

[37]http://www.hec.unil.ch/agoyal/docs/PredictorData2010.xls. This updated data set includes the predictor values through 2010.

[38]http://mba.tuck.dartmouth.edu/pages/faculty/ken.french

[39]The survey data set begins in 1968:Q4. The number of forecasters in the survey varies through time. The median number of forecasters is about 30. For more details in the SPF data, refer to the SPF documentation at http://www.philadelphiafed.org. In addition, there are a number of academic papers utilizing the SPF data set that can be found on their website.

[40]See Dopke and Fritsche (2006), Patton and Timmermann (2010), Patton and Timmermann (2008), and Lahiri and Sheng (2010).

[41]Drechsler (2008) set the ambiguity level to be the variance of the time-varying mean and variance of the consumption growth. Then, the variance premium changes over time according to the ambiguity level. He assumes that the cross-sectional dispersion of SPF's forecasts on the real GDP can be a proxy for the

Choice of Variables. I use two variables from the SPF data: the point forecasts on the real GDP growth and the recession probability. My choice of variables is supported by the assumptions in the model and the informational characteristics of the SPF data. As for the first variable, following the assumption in the model, I conjectured that SPF's point forecasts are refined by an extra signal in addition to the history of the real GDP growth. A simple predictive regression analysis shows that the median or mean of individual forecasts successfully predicts the real GDP growth. In particular, the forecasts on the current quarter and the next quarter outperform a kalman filter or a two-state Markov-switching model with the history of real GDP growth. This implies that the forecasters have more information than the history of real GDP growth.[42] As for the second variable, I choose the recession probability forecasts over the probability range forecasts for two reasons. For one thing, the SPF has changed the form of its questionnaire for the probability range forecasts several times. The main problem is that the probability range assigned into a bin of a histogram has changed. It almost doubled the median of the real GDP growth variance implied by each probability range forecast in the period of 1981:Q3–1991:Q4, which exactly matches the period in which the probability range assigned into a bin is the widest in the history of the SPF. Unfortunately, these changes affected the probability range forecasts by those who were not familiar with the probability distribution or those who were not willing to make an effort to fill all the blanks. Unlike the probability range forecasts, the questionnaire for the recession probability forecasts has not changed since the beginning of the data collection. Fortunately, the recession probability forecasts do not show any obvious evidence of behavioral bias or insincerity on the part of the forecasters.[43] Furthermore, the questionnaire for the the recession probability forecasts is designed to be free of the seasonality which appears to be very strong in the probability range forecasts.

4.2 Testable Implications and Econometric Challenges

The model developed in Section 2 has several interesting implications regarding the conditional equity premium and the cross-sectional variation.

ambiguity level. Bansal and Shaliastovich (2009) utilize the same dispersion to capture the uncertainty in the current time-varying mean of the consumption growth which is proportional to the impreciseness of signals in their model. Anderson et al. (2009) use the dispersion of SPF's forecasts on the real GDP and the corporate profits for the ambiguity level although their theoretical and empirical approaches are different from Drechsler (2008). Ulrich (2010) uses the dispersion of SPF's forecasts on the real GDP and the inflation for the amount of ambiguity.

[42] Ghysels and Wright (2009) provide empirical evidence that financial market data can predict the future professional forecasts. Even though the forecasters do not have more information than the market, the approach in this paper is consitent with their result. I assume that (1) the forecasters are a random sample from the agents in the economy, and (2) they receive an additional signal in addition to the history of real GDP growth.

[43] See Ghysels and Wright (2009) and Lamont (2002) for behavioral biases in forecasters.

⟨**Testable Implications**⟩

1. Variable information quality predicts future excess market returns. If the information quality process is sufficiently persistent, the coefficient in the predictive regression is positive as in Figure 5.

2. The prediction power of information quality becomes stronger when information quality is controlled by uncertainty, especially by prior uncertainty[44] as shown in Figure 6 and 7.

3. Information quality enhances the return predictability of the dividend-price ratio by capturing the other time-variations in the conditional equity premium. Likewise, the dividend-price ratio improves the return predictability of information quality.

4. The model-implied stochastic discount factor explains the cross-sectional variation in expected returns.

However, there are many econometric challenges in performing an empirical study to test these implications, as most variables of interest are econometrically unobservable.

⟨**Econometric Challenges**⟩

1. We do not know the exact dynamics of the economy in an agent's belief. To make matters worse, models and parameters can change through time either in the true economy or only in the agent's belief.[45] This can cause a severe model misspecification problem and build too rigid a structure in an agent belief-updating process.

2. We, as econometricians, cannot directly observe the additional signal, which is an implied aggregate signal during the period rather than actual news in the real world.

3. Information quality that the agent perceives is not observable. Neither is her uncertainty, nor output (or consumption) growth volatility. They are all entangled in data.

4. The pricing model is based on a single representative agent. However, we have multiple forecasters with a heterogeneous belief in survey data. It is critical to find the best way of aggregating information contained in the survey data.

In the next subsection, I will explain how to tackle or avoid these econometric issues. Unlike the existing literature, my approach neither involves arbitrary inclusion of instrumental variables, signal variables, or proxies, nor imposes an arbitrary model structure.

[44]Here, prior uncertainty is $var_{t-1}(\mu_t)$ which is uncertainty before an agent receives a signal for μ_t at time t. Posterior uncertainty is $var_t(\mu_t)$ which is uncertainty after observing the signal.

[45]Johannes et al. (2010) assume that an agent does not know the parameter values. Ozoguz (2009) builds an empirical model in which the transition probability is time-varying.

4.3 Econometric Model

I made several assumptions inevitable for identification and estimation. Some of them will be more refined later, but I start with a simple version.

A1. Professional forecasters are a random sample from the population of the agent $i = 1, ..., I$.

A2. As in the model, all agents believe that they know the current values of the parameters, states, signals, and model, except for μ_t the conditional mean of the next period $(t+1)$ consumption (or output) growth.

A3. An individual agent's prior and posterior belief about μ_t can be approximated by a normal random variable.

A4. Each agent updates her belief following Bayes' theorem and reports her posterior mean as a point forecast.

A5. There is heterogeneity across the agents in the signals they receive, and in the prior and posterior mean of μ_t in each agent's belief. In case there is heterogeneity in other variables, models, or belief-updating process, the econometric approach here can capture their cross-sectional aggregate, which corresponds to those of a representative agent.

A6. Each period, an additional signal $y_{i,t}$ is given to each agent i who believes

$$y_{i,t} \sim \mathcal{N}(\mu_t, q_t^{-1}) \tag{4.1}$$

I recycle a variable q_t from Section 2. It used to denote information quality as a probability of correct indication of the true state of the economy. Hereafter, it still represents information quality, however, as a precision (reciprocal of variance) of the signal which is a normal random variable now. The agent i uses only $y_{i,t}$ to update her belief on μ_t and does not care about other agents' signals. In other words, $y_{i,t}$ in my model is defined as the hypothetical signal that induces this behavior of the agent. Therefore, the assumption is actually about the existence of such hypothetical signals.

Some of the assumptions in the econometric model are different from those in the economic model in Chapter 2. In assumption A5, I allow heterogeneity across the agents since any survey data like the SPF reveals a disagreement among the forecasters. In assumption A6, a signal variable is a normal random variable rather than a Bernoulli random variable defined in the model in Chapter 2. I will explain these differences in assumption after defining other variables in the econometric model.

In the SPF data, we can directly observe the point forecasts and recession probability by each individual forecaster, as well as the cross-sectional forecast dispersion. In addition to these variables, I define the other variables hidden to an econometrician as follows:

- Risk, $v_{i,t}$: the conditional variance of output or consumption[46], i.e., $var_{i,t}(\log \frac{C_{t+1}}{C_t}|\mu_t)$.
- Implied Individual Signal, $y_{i,t}$: as defined in assumption A5.
- Implied Aggregate Signal, y_t: aggregate counterpart of $y_{i,t}$ across agents. This is a hypothetical signal to the representative agent.
- Quality of Implied Individual Signal, $q_{i,t}$: precision (or reciprocal of variance) of $y_{i,t}$. This will be referred to hereafter as (absolute) information quality.
- Prior Individual Uncertainty, $u_{i,t}$: the variance (implied by posterior belief of an agent) of the conditional mean growth rate at the next period before she observes a signal $y_{i,t}$, i.e., $var_{i,t-1}(\mu_t)$.
- Posterior Individual Uncertainty, $u^*_{i,t}$: the variance (implied by posterior belief of an agent) of the conditional mean growth rate at the next period after she observes a signal $y_{i,t}$, i.e., $var_{i,t}(\mu_t)$.
- Total heterogeneity in signals and everything else, h_t: this is the variance of the residuals of all the variations across the agents' belief-updating processes after we extract the hypothetical representative agent's belief-updating process from the data.

According to Assumption A5, if variables such as $v_{i,t}$, $q_{i,t}$, $u_{i,t}$, and $u^*_{i,t}$ are the same across the agents then we can drop the subscript i. If not, the econometric approach here can still estimate the cross-sectional aggregate of those variables, and the estimated aggregate variables are assumed to be those of the representative agent.

In a single representative model, there could be uncertainty but no heterogeneity. It assumes the whole market acts like a single agent. To specify this intuition behind the single agent model, I construct market-wide uncertainty that the single representative agent perceives as follows:

[46] I set $v_{i,t}$ to be the volatility of output growth. However, there is no difference between output and consumption growth in the economic model.

- Prior Market Uncertainty $U_t = u_t + h_t$: this is the sum of Prior Individual Uncertainty and Total heterogeneity. It is assumed to measure the hypothetical prior uncertainty of the representative agent in the pricing model.
- Posterior Market Uncertainty $U_t^* = u_t^* + h_t$: this is the sum of Posterior Individual Uncertainty and Total heterogeneity. It is assumed to measure the hypothetical posterior uncertainty of the representative agent in the pricing model.

This summation is technically plausible since both terms are variance terms, so that the resulting term is also a variance term.[47] Of course, this is not the only way to construct uncertainty in a single agent model from the multi agent data. The most common way is simply to use a proxy: cross-sectional forecast dispersion. The new approach in this paper seems more complicated; however, this is a minimal structure among the modelsdifferentiating (or controlling) all the variables such as prior and posterior uncertainty, information quality, and implied aggregate signal. The benefit of this approach is that it enables us to perform a simultaneous estimation of those variables while avoiding overuse of the proxy.

- Prior Relative Information Quality $Q_t = q_t/U_t$: In learning-based models, uncertainty is the main (unique in most cases) state variable in the representative agent's continuation value. To focus on the pure effect of information quality, I define Relative Information Quality as information quality (precision) divided (controlled) by Prior Market Uncertainty.
- Posterior Relative Information Quality $Q_t^* = q_t/U_t^*$

In fact, Prior and Posterior Relative Information Quality play an similar role but I primarily use the former in empirical analysis because Figure 6 and 7 show the former $Q_t = q_t/U_t$ has clearer implications in predicting returns. Besides, U_t^* is a function of q_t, so a control of q_t by U_t^* make less sense than by a control by U_t.

Following the notation in $u_{i,t}$ and $u_{i,t}^*$, I refine the point forecasts observed from each forecaster as follows:

- Prior Individual Forecast $\hat{\mu}_{i,t}$: the mean (implied by posterior belief of an agent) of the conditional mean growth rate at the next period before she observes a signal $y_{i,t}$, i.e., $E_{i,t-1}[\mu_t]$.
- Posterior Individual Forecast $\hat{\mu}_{i,t}^*$: the mean (implied by posterior belief of an agent) of the conditional mean growth rate at the next period after she observes a signal $y_{i,t}$, i.e., $E_{i,t}[\mu_t]$.

[47] Market-wide uncertainty is probably a function of the average individual uncertainty and heterogeneity between agents, though its form is unknown.

By assumption A3 and A4, the belief-updating process of an agent at each time t can be summarized as

$$\hat{\mu}^*_{i,t} = \frac{u^{-1}_{i,t}\hat{\mu}_{i,t} + q_{i,t}y_{i,t}}{u^{-1}_{i,t} + q_{i,t}} \tag{4.2}$$

$$\Phi^{-1}(1 - Pr_{i,t-1}[recession_{t+1}]) = \frac{\hat{\mu}_{i,t}}{\sqrt{u_{i,t} + v_t}} \tag{4.3}$$

$$\Phi^{-1}(1 - Pr_{i,t}[recession_{t+1}]) = \frac{\hat{\mu}^*_{i,t}}{\sqrt{u^*_{i,t} + v_t}} \tag{4.4}$$

where $u^*_{i,t} = (u^{-1}_{i,t} + q_{i,t})^{-1}$, $\Phi(\cdot)$ is a cumulative distribution function of standard normal distribution, and $Pr_{i,t}[recession]$ is a reported recession probability[48] from the SPF data. We observe $\hat{\mu}_{i,t}$ and $\hat{\mu}^*_{i,t}$ from the SPF data as well. Then, I make Equation (4.2), (4.3), and (4.4) be the likelihood function by adding heterogeneity terms with h_t. The volatility is identified from the likelihood of the observed output growth. As a result, we have four likelihood functions at each time t. Nevertheless, it is not easy to estimate all the time-varying hidden variables without a dynamic structure. The number of unknowns is much larger than the number of observations. To tackle this identification problem, I impose a dynamic structure of those hidden variables as in a Kalman filter. I assume that y_t and log of u_t, v_t, h_t, and q_t follow AR(1) process. Then, I perform a nonlinear filtering as described in Section 5. The obtained series is a filtered series which is estimated using information up to time t. So, we are free of look-ahead bias in a predictive regression when we use these series as a predictor.

There is an important reason why log AR(1) processes are more suitable for estimation while the pricing model assumes a two-state Markov-switching process.[49] First, the pricing model should have a jump component to model a fluctuation of the uncertainty. A two-state Markov-switching process is one of the simplest forms with a jump component. Thus, modelling with such a process helps us understand important pricing implications clearly without unnecessary ornaments. However, it can be problematic to directly apply a two-state Markov-switching process in an estimation. Forecasts from a two-state Markov-switching

[48] Recession probability is defined as negative real GDP growth in the questionnaire in the SPF.

[49] Log AR(1) processes can quickly respond to a sudden jump, compared to a discrete version of CIR process. Also, Log AR(1) processes do not entail negative value particles in simulation step. Although we can always add an additional element such as a jump component to fit the data better, it turns out that log AR(1) process is enough to capture the time-variation of hidden variables.

model should have an upper and a lower boundary. Also, the unconditional distribution of forecasts should be bimodal at both boundaries. Real forecast data is distributed like a normal random variable. Furthermore, it is an obvious misspecification that the high state should be higher than the historical maximum value. Since the values of high and low state are crucial in determining belief-updating behavior, such misspecification can cause severe problems in an estimation. Another issue is, as emphasized in Ozoguz (2009), that the transition probability is likely to be time-varying. Unfortunately, belief-updating behavior is also heavily affected by the values of the transition probability. If there is a huge gap between the true probability value and the one we use, the estimation result will be totally unreliable. My setup of filtering likelihood is more robust to the change in parameters or underlying models. Since the likelihood function only captures an in-between behavior of the agent, the filtering algorithm naturally avoids the misspecification problem that can be caused by structural changes. The same logic can be applied to the case of a signal variable as well. Yae (2011a) compares different specifications at a glance and illuminates the benefit of the approach pursued in this paper. A sequential parameter estimation and smoothing methods are also developed.

5 Estimation Methods

5.1 Sequential Inference for Hidden States and Parameters

By Bayes' theorem, the posterior distribution of the parameter θ conditioned on the observation y is defined as

$$p(\theta|y) = \frac{p(y|\theta)p(\theta)}{p(y)} \propto p(y|\theta)p(\theta)$$

where $p(y|\theta)$, $p(\theta)$, and $p(y)$ are the likelihood, the prior distribution, and the marginal likelihood, respectively. In our model, the likelihood $p(y|\theta)$ is defined as

$$p(y|\theta) = \int p(y|\theta, X_{1:t})p(X_{1:t}|\theta)dX_{1:t} \tag{5.1}$$

which is not known in an analytic form. Alternatively, we can explore the joint posterior distribution of the parameter θ and the latent states $X_{1:t}$ conditioned on observation y, which is defined as

$$p(\theta, X_{1:t}|y) = \frac{p(y|\theta, X_{1:t})p(\theta, X_{1:t})}{p(y)} \propto p(y|\theta, X_{1:t})p(\theta, X_{1:t}) \tag{5.2}$$

where $p(y|\theta, X_{1:t})$ and $p(\theta, X_{1:t})$ are the augmented likelihood and the joint prior distribution, respectively. The marginal posterior distribution of the parameter and the latent states is derived from the joint posterior distribution.

$$p(\theta|y) = \int p(\theta, X_{1:t}|y)dX_{1:t} \quad \text{and} \quad p(X_{1:t}|y) = \int p(\theta, X_{1:t}|y)d\theta \tag{5.3}$$

5.2 Particle Filtering and Parameter Learning

The particle filter is a Monte Carlo counterpart of the Kalman filter. Both filters provide the filtering distributions[50] of the latent state variables and the value of the integrated likelihood (5.1) for fixed parameters. While the Kalman filter can be used only for a linear state space model with Gaussian errors, the particle filter generally can handle a non-linear state space model with non-Gaussian errors at the cost of acceptable Monte Carlo errors.[51] As a by-product, a particle filter can approximate the integrated likelihood (5.1) which is necessary for calculating DICor Bayes factor.

Furthermore, the particle filter can be extended to an alternative to MCMCavoiding particle degeneracy of the parameters. Liu and West (2001) and Gilks and Berzuini (2001) sequentially approximate a posterior distribution of the static parameters by adding an artificial noise or MCMC resample-move step. While these approaches can be generally applicable, it is unattractive to a user that the result and the efficiency vary substantially depending on the detail of those 'jittering' steps. Another approach studied in Storvik (2002) and Johannes and Polson (2007) utilizes sufficient statistics.For some special cases, it is known that a sequential sufficient statistics structure exists for the static parameters. Thus, the parameter particle can be easily re-sampled and even Rao-Blackwellization is possible. A simple version of a particle filtering and parameter learning algorithm is given by

Step 1: Initialize particles. For $g = 1, .., G$, draw $(\theta^{(g)}, X_0^{(g)}) \sim p(\theta, X_0)$

Step 2: For $t = 1, .., T$,

a) Propagate states. For $g = 1, .., G$, draw $X_t^{(g)} \sim p(X_t|X_{t-1}^{(g)}, \theta^{(g)})$

b) Resample particles with the weight $w^{(g)} \propto p(Y_t|X_t^{(g)}, \theta^{(g)})$

c) Update sufficient statistics. For $g = 1, .., G$, Set $S_t^{(g)} = \mathcal{S}(Y_t, X_{(t-1):t}^{(g)}, S_{t-1}^{(g)})$

b) Resample parameters. For $g = 1, .., G$, draw $\theta^{(g)} \sim p(\theta|S_t^{(g)})$

The algorithm for a particle filter with fixed parameters is obtained by omitting steps 2c and 2d.

[50] For smoothing distribution, see Godsill et al. (2004) with a pure state smoothing case, and Carvalho et al. (2007) with a combined parameter estimation case.

[51] A variant of the algorithms has been developed to reduce Monte Carlo errors in the particle filter and to overcome well-known sample impoverishment. See for example Pitt and Shephard (1999), Doucet et al. (2001), Liu (2001), and Johannes et al. (2009c).

6 Empirical Results

The timing of the Survey. With the survey data up to the quarter of t, a nonlinear filter (particle filter) sequentially estimates variables at the quarter t: information quality, uncertainty, implied aggregate signal, volatility, and heterogeneity. However, it is tricky to determine the timing of the information set because of the uncertain deadline for submissions by the forecasters. According to the SPF document, the deadline for submissions has been the midpoint of each quarter since the third quarter of 1990. The Federal Reserve Bank of Philadelphia conjectures that the deadline in the period before the third quarter of 1990 was not very different. For the cross-sectional analysis, I assume that the reports from professional forecasters reflect their belief at one month before the end of each quarter. For example, I reconstruct the quarterly return data from the monthly returns so that the forecasts in the second quarter correspond to the asset return from February to May. Thus, the cross-sectional regressions analyze the relation between these reconstructed quarterly returns and the filtered variables with the survey data. For the predictive regressions, I keep the timing of the standard quarterly asset returns. Thus, if the deadlines have been met all the time, the predictive regressions do not include the returns of one and a half month after the forecasts are reported. Unlike the cross-sectional analysis, the predictive regression result usually has small R^2, especially if the prediction horizon is short.[52] Thus, the return prediction result can be falsely significant only if there is a slight overlap between the return prediction period and the survey period.[53] For this reason, I exclude the return data of one and a half month as a buffer for possible contamination of the survey data due to the timing of forecast submission.

Filtering Result. In the filtering problem defined in Section 4, there are five latent state variables: information quality, uncertainty, implied aggregate signal, volatility, and heterogeneity. Each state variable follows a log AR(1) process, and they are independent of one another. I set the prior distribution of these latent state variables except for volatility to be an implied stationary distribution of the specified time-series process. The prior distribution of volatility is obtained from the posterior distribution of a linear filtering problem, using real GDP growth data from 1950:Q1 to one quarter before the beginning of the survey data 1968:Q4. The particle filtering and learning algorithm in Section 5 can sequentially estimate the states and parameters at the same time. However, I simplify the empirical analysis by using fixed parameters which are reasonable in terms of the observed output dynamics. The

[52]The log dividend price ratio and *cay* only show R^2 of 3.5% when they are used together to predict the quarterly excess market returns.

[53]Forecasters' beliefs can be heavily affected by asset prices from the financial market, as implied by Andreou et al. (2010) and Ghysels and Wright (2009).

results are robust to the different parameter values. Unlike the learning procedure in Johannes et al. (2010), the filtering procedure of this paper is used for the purpose of tracking the latent state variables rather than estimating parameters. Parameter value itself is neither effective nor important in the empirical analysis of this paper, which confirms the model implications qualitatively, not quantitatively. Yae (2011a) provides an extensive empirical study for sequential parameter estimation and model averaging to improve return predictability.

Figure 11 shows the filtered distributions of various uncertainty measures defined in Section 4. During the last financial crisis, all uncertainty measures increased, especially *prior individual uncertainty*, which almost tripled from the period before the crisis, recording the historical high during the sample period from 1968:Q4 to 2011:Q3. Unlike *prior individual uncertainty*, *market uncertainty* increased in the period of the Nasdaq bubble. Figure 12 displays the filtered distributions of various information quality measures defined in Section 4. Information quality changes over time persistently. In some periods, big hikes and drops are shown, implying that a jump process is related.[54] When the filtered information quality is fitted to AR(1), the persistence coefficient is about 0.8, which is sufficiently persistent to generate the same asset pricing implication as in Section 2. Figure 13 displays interesting by-products of the filtering procedure. In figure 13(a), the forecasters do not perceive particulary lower signal precision for bad news than for good news, counter to the assumption in Epstein and Schneider (2008).[55] Figure 13(b) shows the forecasters do not consider extremely good or extremely bad output growth as a reliable signal.[56] In Figure 13(c) and 13(e), information quality and *prior market uncertainty* are positively correlated ($\rho = 0.22$, t=3.2), and shocks in their dynamics are highly correlated ($\rho = 0.43$, t=3.8), implying that *prior market uncertainty* induces higher information quality.[57] However, higher information quality does not induce lower *prior market uncertainty* at the next period, as shown in Figure 13(f). In the model in Section 2, this positive correlation between information quality and *prior uncertainty* yields stronger return predictability of information quality.

[54]It is possible to incorporate a jump process into a log AR(1) process to improve the fitting of the data. However, the filter without a jump component still works fine in tracking the latent state variables. The conclusion of this paper remains the same.

[55]This does not necessarily mean the assumption in Epstein and Schneider (2008) is flawed. Since I estimated hypothetical aggregate signals, actual signals to agents can be different. Besides, it might be merely because the forecasters use an objective measure on signal precision when they report their forecasts, while they stick to a distorted measure when they make an investment decision.

[56]This is consistent with the learning model with a signal whose precision is unknown. See Subramanyam (1996) and Hess et al. (2010).

[57]Due to this positive correlation, higher information quality does not necessarily imply lower *posterior market uncertainty* as shown in Figure 13(d), as opposed to the standard learning model with independent shocks. These results are also consistent with the setting that there are pure uncertainty shocks and agents endogenously acquire information according the level of uncertainty they face.

Predicting Excess Market Return. For return predictive regression, I use the sequentially filtered variables as a predictor, avoiding a look-ahead bias from using a pre-constructed proxy.[58] The full sample period of the SPF is from 1968:Q4 until 2011:Q3. However, I have removed the first 16 quarter observations to mitigate the effect of the priors. The data used in predictive regressions are from 1973:Q1 to 2010:Q4. Table 4 compares the return predictability of different information quality and uncertainty measures obtained by a nonlinear filter. Among other variables, only information quality strongly predicts quarterly excess market returns. Consistent with model implications, information quality shows better predictability when controlled by *prior market uncertainty* U_t in Table 4(B). Unlike information quality, uncertainty does not show strong predictability, which is consistent with the result by Ozoguz (2009).[59] Table 5 shows return predictability of a various nonlinear transformation of signal precision. Since I do not derive an exact quantitative relation between the conditional equity premium and information quality, predictive regression results can depend on the specific form of information quality. Columns from (1) to (4) showthat information quality strongly predicts future excess returns regardless of its nonlinear transformation. One standard deviation increase in information quality predicts a $2.0 \sim 3.5\%$ increase in quarterly excess market returns with R^2 of $4.8 \sim 6.5\%$, depending on the specifications.

A more interesting result is shown in Table 5(B). Information quality is not just orthogonal to dividend price ratio and *cay*, but the three of them perform better when they are used altogether as predictors. R^2 of regression with dividend price ratio and *cay* is 3.5%, but it increases to 8.8% when information quality is included as an additional predictor. R^2 of 8.8% is larger than the sum of each regression R^2 of 4.8%+3.5%=8.3%. This can happen when information quality acts like a control variable. In such cases, the coefficient, t-statistic, and R^2 all can rise. This is exactly what Figure 8 implies.

The return prediction result of *prior relative information quality* (hereafter Q_t) is even morestriking. Table 6(B) shows that Q_t neutralizes *cay* but boosts predictability of the dividend price ratio. Besides, predictability of information quality still remains the same or becomes stronger when fourteen other predictors from Goyal and Welch (2008) are included together in a multiple regression, as shown in Tables 5(C) and 6(D). In longer horizon prediction, Q_t outperforms (absolute) information quality q_t as shown in Table 7. R^2 of the predictive regression by (absolute) information quality starts decreasing over four quarter horizons and almost disappears at three year horizons. However, R^2 of the predictive regres-

[58]Brennan and Xia (2005) attribute return predictability of *cay* to a look-ahead bias.

[59]Johannes et al. (2010) report that the model-implied dividend price ratio predicts return especially when parameters are sequentially estimated. This can be taken as evidence for time-varying parameters. Boguth and Kuehn (2009) report future excess return is predicted by the change of consumption volatility in an agent's belief.

sion by Q_t keeps increasing until two year horizons and remains high until three year horizons. Table 7(D) and Figures 14 and 15 show that the dividend price ratio and Q_t perform very well when they work together. R^2 is 21.5% for one year horizon and 37.9% for two year horizon. Table 8 shows the prediction results for two sub-periods. The return predictability of Q_t is robust to the sample period, although the predictability is stronger at the first half (1973:Q1-1991:Q1) of the whole period. All other predictors and (absolute) information quality are relatively struggling in the second half (1991:Q2-2010:Q4).

Furthermore, Q_t outperforms all other predictors from Goyal and Welch (2008) in the out-of-sample prediction in terms of R^2, as shown in Table 9.[60] Among four different nonlinear forms of Q_t, the two forms $-Q_t^{-1}$ and $-Q_t^{-2}$ predict excess market return with positive out-of-sample R^2 regardless of prediction horizon and period. Even the worst one among the four, Q_t, outperforms all other predictors from Goyal and Welch (2008). Out-of-sample R^2 is up to 6.1% for one quarter horizon and 17.2% for one year horizon.

Cross-sectional Variation. The model developed in this paper has a stochastic discount factor which is a function of three state variables: information quality, prior uncertainty, and aggregate signal.[61] However, an analytic form of the stochastic discount factor is not available. Thus, I instead show a potential performance of the asset pricing model in Section 2 by constructing a linear factor model with these state variables in the stochastic discount factor:

$$R^e_{i,t} = \beta_q \Delta \log q_t + \beta_u \Delta \log U_t + \beta_y y_t + \epsilon_{i,t}$$

where q_t is information quality, U_t is prior market uncertainty, and y_t is implied aggregate signal. This three-factor model explains the cross-sectional variation in expected returns. Table 10 shows the two-stage cross-sectional test result.[62] I include ten Size-sorted, ten Book-to-Market-sorted, and ten Momentum-sorted portfolios as test assets, following Ozoguz (2009). As they show, uncertainty factor can solely explain a considerable amount of variations (R^2=53.2%). Furthermore, my three-factor model has higher cross-sectional R^2 and lower root mean squared errors than Carhart's Four-Factor Model. However, the market prices of risk are not always significant, possibly due to the small sample size or time-varying betas. Figure 16 depicts pricing errors by several linear factor models. Momentum-sorted portfolios are well priced in all figures. Evidence of pricing the other portfolios is moderate in all cases.

[60] Out-of-sample R^2 (percentages) is calculated as $R^2 = (1 - var(\hat{e}_t)/var(R^e_t)) \times 100$, where $\hat{e}_t$ is a forecasting error and R^e_t is the excess market return in the prediction period. The regression coefficients are estimated at every quarter t using the data from 1973:Q1 up to time t to predict R^e_{t+1}.

[61] The stochastic discount factor can be summarized by the two components: information quality and posterior uncertainty.

[62] This equals Fama-MacBeth regression with constant betas except for standard errors of market price of risk.

7 Discussions

Outside Validation A potential criticism to the empirical approach of this paper is whether the obtained result can be confirmed by an outside validation. In the literature on uncertainty at the macroeconomic level, information quality has been barely measured. Bansal and Shaliastovich (2009) measured uncertainty as cross-sectional forecast dispersion but did not differentiate uncertainty from information quality. In the literature on analysts' forecasts, the quality of signals about the firm fundamental has often been measured by cross-sectional forecast dispersion among analysts. These proxies cannot be used for outside validation because they are defined and included as separate entities different from information quality in the econometric model structure of this paper. Thus, the estimated information quality, as defined in this paper, is hard to confirm by other proxies known so far.

On the other hand, information quality in other learning-based models or in a layman's mind can be very different from what is defined and estimated in this paper. The definition and estimation of information quality will depend on how information quality dynamically interacts with uncertainty. Thus, there is still an issue of interpretation, although the pricing model and econometric approach in this paper are designed with a minimal structure in order to be robust to model misspecficaion. For this reason, even if there is a proxy validating the quality of information estimated here, it does not necessarily provide an additional justification of the results of this paper. Rather, empirical performance of the estimated information quality will be a reliable standard to evaluate the pricing model and the econometric approach.

Prediction Performance Forecasting performance is a main concern for practitioners in the financial industry. Although this paper shows very strong return prediction results with high R^2, there still remains considerable room for improvement in the prediction performance. Since a nonlinear filter is used to estimate information quality, it is convenient to customize the likelihood functions and the dynamic structure of hidden state variables. For example, I estimate information quality variable solely from the survey data and the real GDP growth. Thus, return predictability can be improved by adding likelihood functions of the predictive regression into the nonlinear filter. Time-varying coefficients and volatility are another effective extension.[63] To attenuate an outlier effect, a heavy-tailed distribution or quantile rule can be utilized following Johannes et al. (2009b). Also, a jump process can be easily intro-

[63] Using MCMC, Jostova and Philipov (2005) provide a Bayesian estimation approach to time-varying coefficients.

duced to return, volatility, and hidden state dynamics.[64] To avoid an overfitting problem, the optimal level of econometric structure can be naturally chosen by sequential Bayesian model comparison or averaging. All these variations and extensions can be implemented in a single econometric framework, as studied in Yae (2011a).

8 Conclusion

Beliefs of investors play an important role in the financial market. Thus, investors' belief-updating process, obviously, is important as well. Since uncertainty and information are two main driving forces of belief-updating process, asset prices are possibly entangled with uncertainty, information, and their interaction. This paper empirically investigates their complicated relations, focusing on the role of time-varying information quality in the model with ambiguity aversion to signals.

From the survey data, I simultaneously extract time-varying information quality, uncertainty, and the aggregate news. I find that variable information quality strongly forecasts the future excess market returns as predicted by the model. Uncertainty-controlled information quality shows better predictability and improves the return-predictability of the dividend price ratio, which is consistent with the model implication. Evidence on cross-sectional implication is promising and worth studying in more depth. Other empirical evidence from the estimation procedure is also interesting.

This paper tells a story about variability of information quality and an investor's uncertainty. The story rangesfrom a new pricing model to empirical evidence, through a novel application of a particle filter into the survey data. I suggest that this is not just an isolated insight, but rather the beginning of a new area of research.

On the theoretical side, the effect of different types of ambiguity aversion should be studied in more detail to unveil the underlying mechanism thoroughly. So far, there is not much experimental evidence or research on pricing models comparing different types of ambiguity aversion in a unified framework.[65] Model implication for other assets should be studied as well. Uncertainty and information quality about inflation will affect bond prices, variance premium, and derivative prices.On the empirical side, the econometric approach and the empirical results in this paper suggest that survey data can tell much more about investors' belief than what has been studied so far. Now an econometrician can take a similar approach with a data set, such as I/B/E/S or Management Forecasts, to answer other interesting questions. Furthermore, return predictability of information quality should be studied more

[64] This will be a sequential estimation version of Eraker et al. (2003).

[65] This paper utilizes a generalized recursive smooth ambiguity preference proposed by Ju and Miao (2012).

to see whether it is practically meaningful as well and whether it will affect fund managers' portfolio decisions.

Figure 1: **Equity Premium and Ambiguity Aversion** The horizontal axis is ambiguity aversion parameter η, so larger value of η means more ambiguity aversion of an agent. The vertical axis is annualized expected excess market returns with percentage unit. The black solid line shows the conditional equity premium at the high information quality state while the dashed line displays the conditional equity premium at the low information quality state. The gray solid line in the middle shows unconditional equity premium. The preference parameters used in this figure are risk averse parameter $\gamma = 2$, elasticity of intertemporal substitution (EIS) $\rho^{-1} = 1.5$, and subjective discount rate $\beta = 0.973$. I set $p_{ii} = p_{aa} = 0.9$, $q_A = 0.95$, and $q_I = 0.55$ as the third case in Table 1 and the other parameters are from Parameter Set II in Table 2.

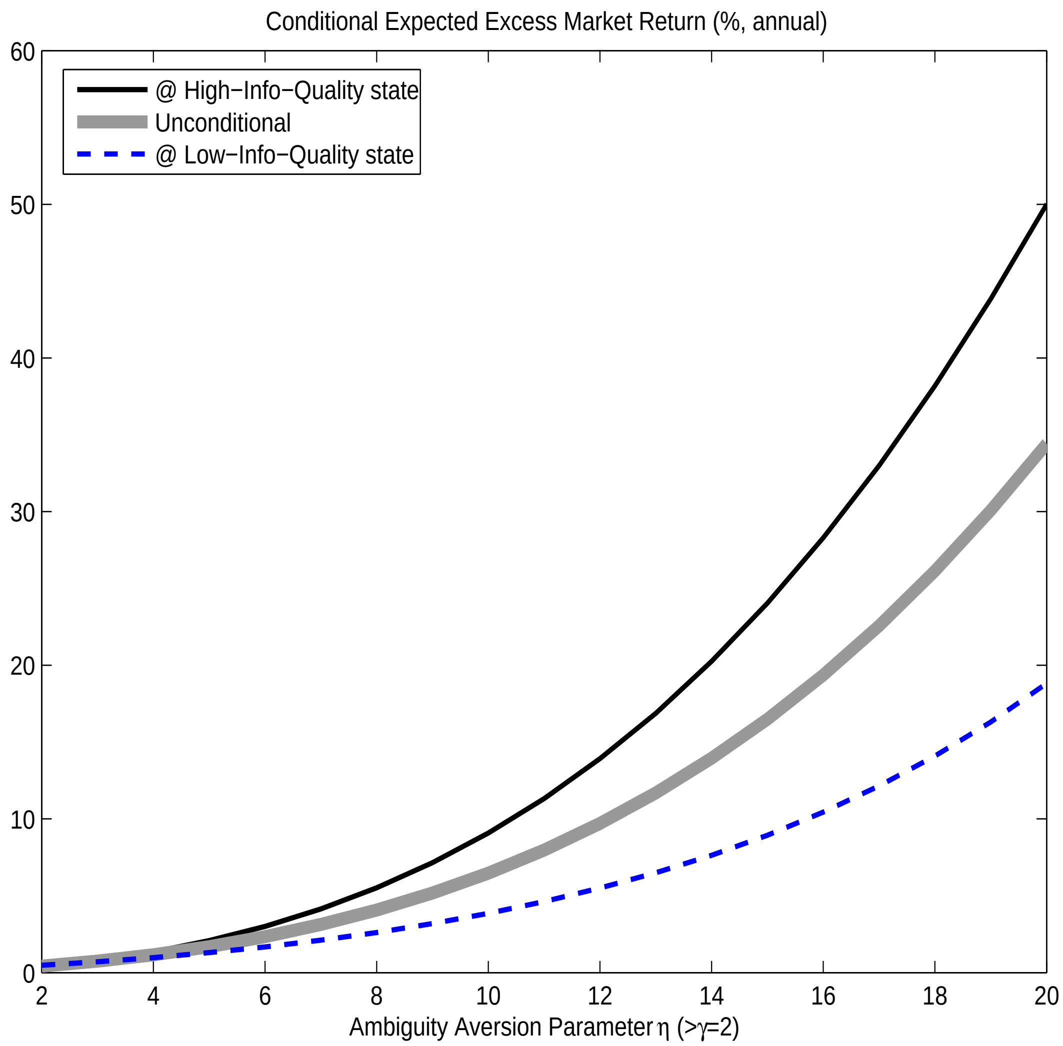

Figure 2: **Decomposition of Unconditional Equity Premium** The figure plots decomposition of the unconditional equity premium which is the gray solid line in Figure 1. As in the legend in the figure, Lines (3)$\sim$(5) show the unconditional equity premium without additional signals by (5) CRRA($\gamma = \eta = \rho = 2$), (4) power utility with ambiguous aversion to signals ($\gamma = \rho = 2, \eta = 8.9$), and (3) recursive utility with ambiguous aversion to signals ($\gamma = 2, \rho = 1.5, \eta = 8.9$). Lines (1) and (2) have the same preference as Line (3) but they are different in the quality of signals and the dynamics of the quality. Line (2) uses constant information quality $q_t = q = 0.75$ and shows the effect of higher information quality. Line (1) repeats the unconditional equity premium in Figure 1, including the term accounting for the investor's desire to hedge changes in information quality. All lines use Parameter Set II in Table 2.

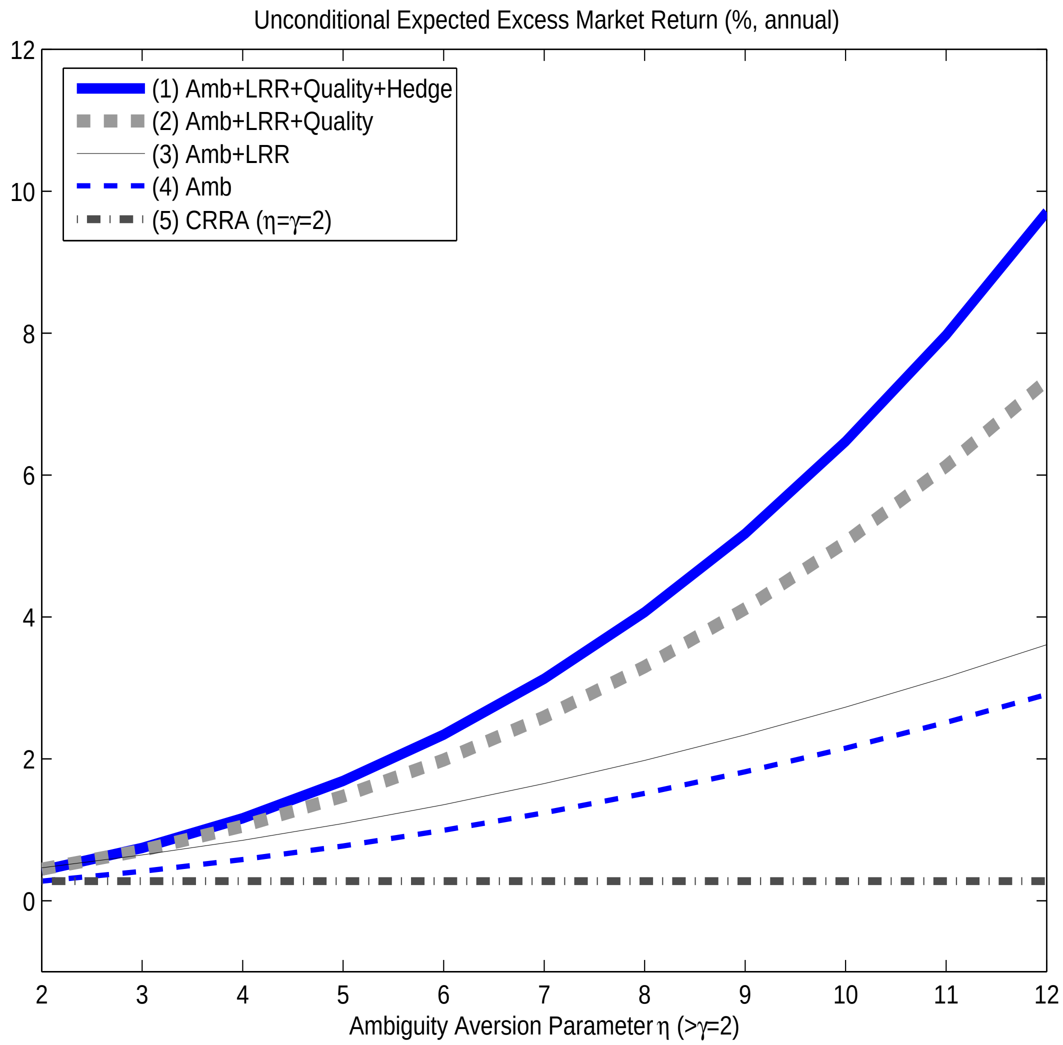

Figure 3: **Conditional Equity Premium versus Information Quality.** Information quality q_t in the horizontal axis represents the probability that the signal y_t (a binomial random variable) correctly indicates true state of the economy. Information quality $q_t = q$ is a constant in this case. The preference parameters used in this figure are risk averse parameter $\gamma = 2$, elasticity of intertemporal substitution (EIS) $\rho^{-1} = 1.5$, and subjective discount rate $\beta = 0.973$. The other parameters are from Parameter Set II in Table 2. 2

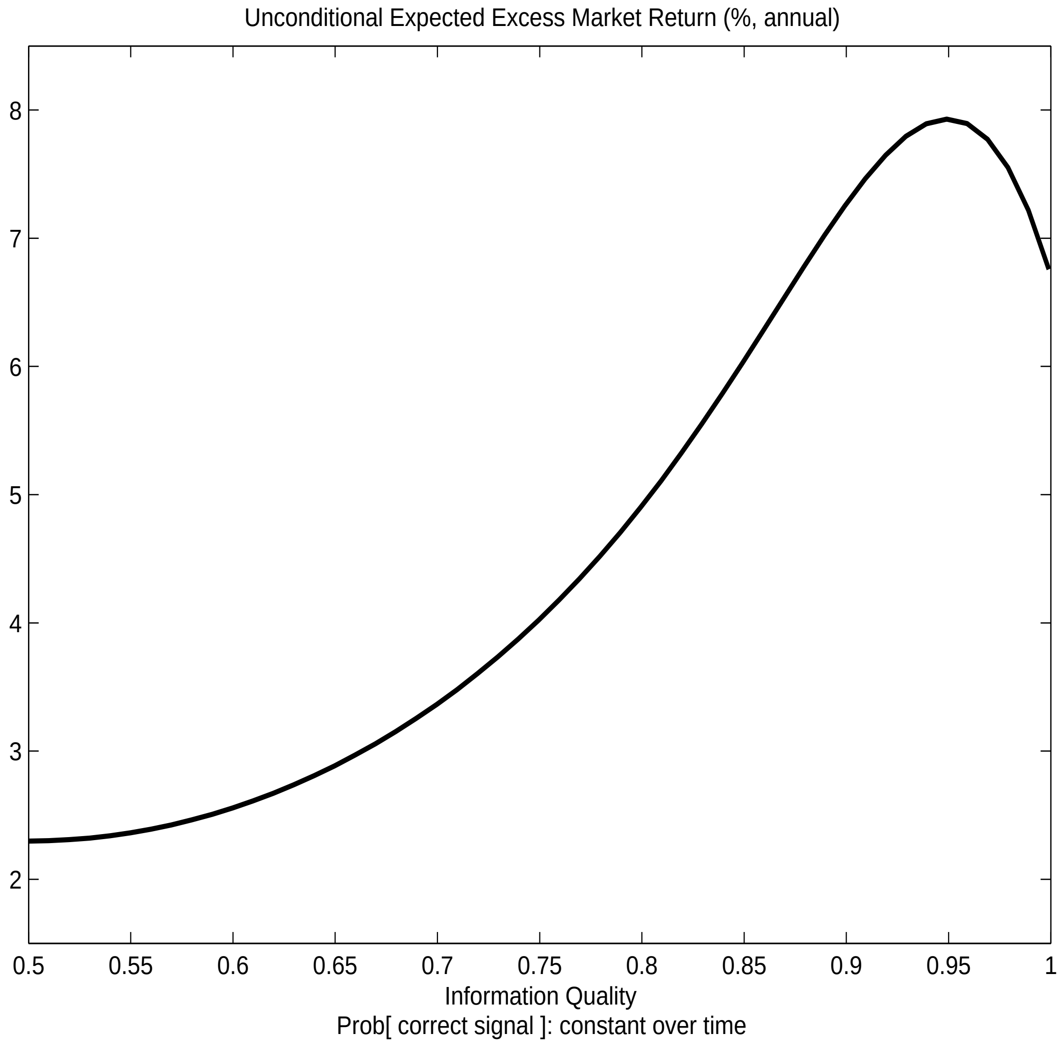

Figure 4: **Conditional Equity Premium versus Time-variability of Information Quality.** The horizontal axis is the probability that the signal y_t correctly indicates the true state of the economy when information quality is high, that is q_A as defined in Section 2. In the figure, I set $q_I = 1.5 - q_A$ so that $E[q_t] = 0.75$. Therefore, larger value of q_A means more time-variability of information quality while the average information quality remains the same. The vertical axis is annualized equity premium with percentage unit. The black solid line shows the conditional equity premium at the high information quality state while the dashed line displays the conditional equity premium at the low information quality state. The gray solid line in the middle shows unconditional equity premium. The preference parameters used in this figure are risk averse parameter $\gamma = 2$, elasticity of intertemporal substitution (EIS) $\rho^{-1} = 1.5$, and subjective discount rate $\beta = 0.973$. I set $p_{ii} = p_{aa} = 0.9$ as the third case in Table 1 and the other parameters are from Parameter Set II in Table 2.

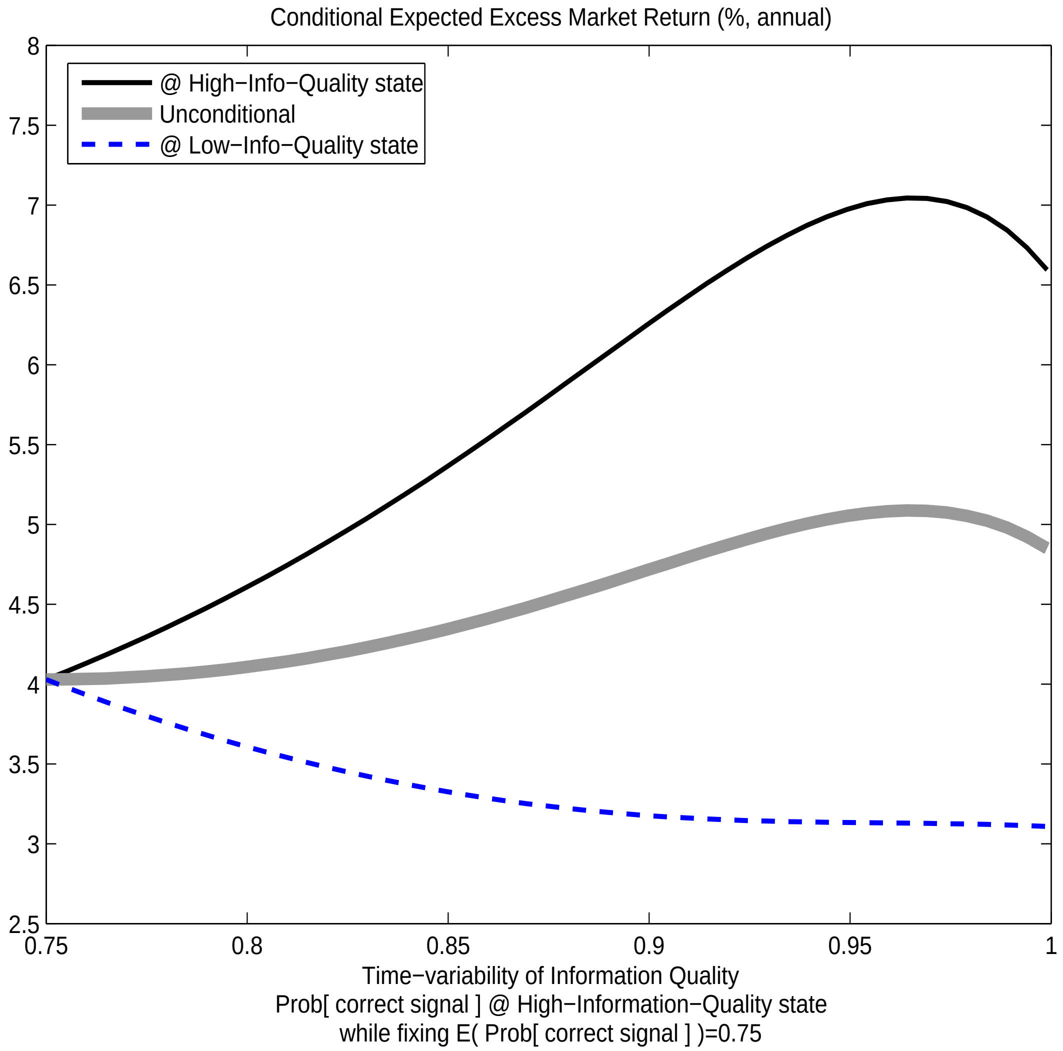

Figure 5: **Conditional Equity Premium versus Persistence of Information Quality.** The horizontal axis is the probability that information quality remains at the same state, that is $p_{aa} = p_{ii}$ as defined in Section 2. In the figure, larger value of p_{aa} means more persistence in dynamics of information quality. The vertical axis is annualized conditional equity premium with percentage unit. The black solid line shows the conditional equity premium at the high information quality state while the dashed line displays the conditional equity premium at the low information quality state. The gray solid line in the middle shows unconditional equity premium. The preference parameters used in this figure are risk averse parameter $\gamma = 2$, elasticity of intertemporal substitution (EIS) $\rho^{-1} = 1.5$, and subjective discount rate $\beta = 0.973$. I set $q_A = 0.95$ and $q_I = 0.55$ as the third case in Table 1 and the other parameters are from Parameter Set II in Table 2.

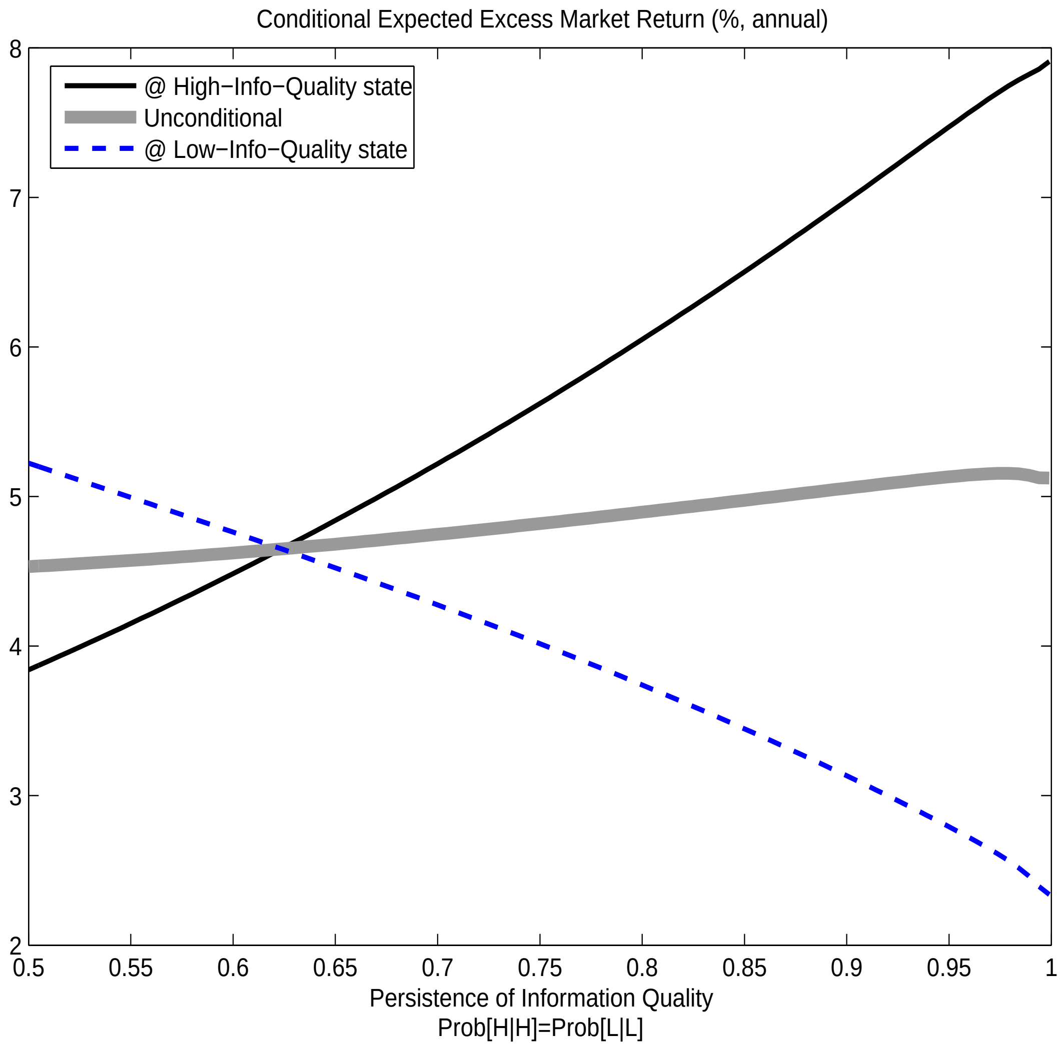

Figure 6: **Conditional Equity Premium versus Posterior Uncertainty.** The vertical axis is annualized conditional equity premium with percentage unit. The horizontal axis is posterior uncertainty which is the variance of μ_t in an agent's belief after she receives a signal y_t as defined in Section 2. The solid line shows the conditional equity premium at the high information quality state while the dashed line displays the conditional equity premium at the low information quality state. The same parameter values are used as Figure 1.

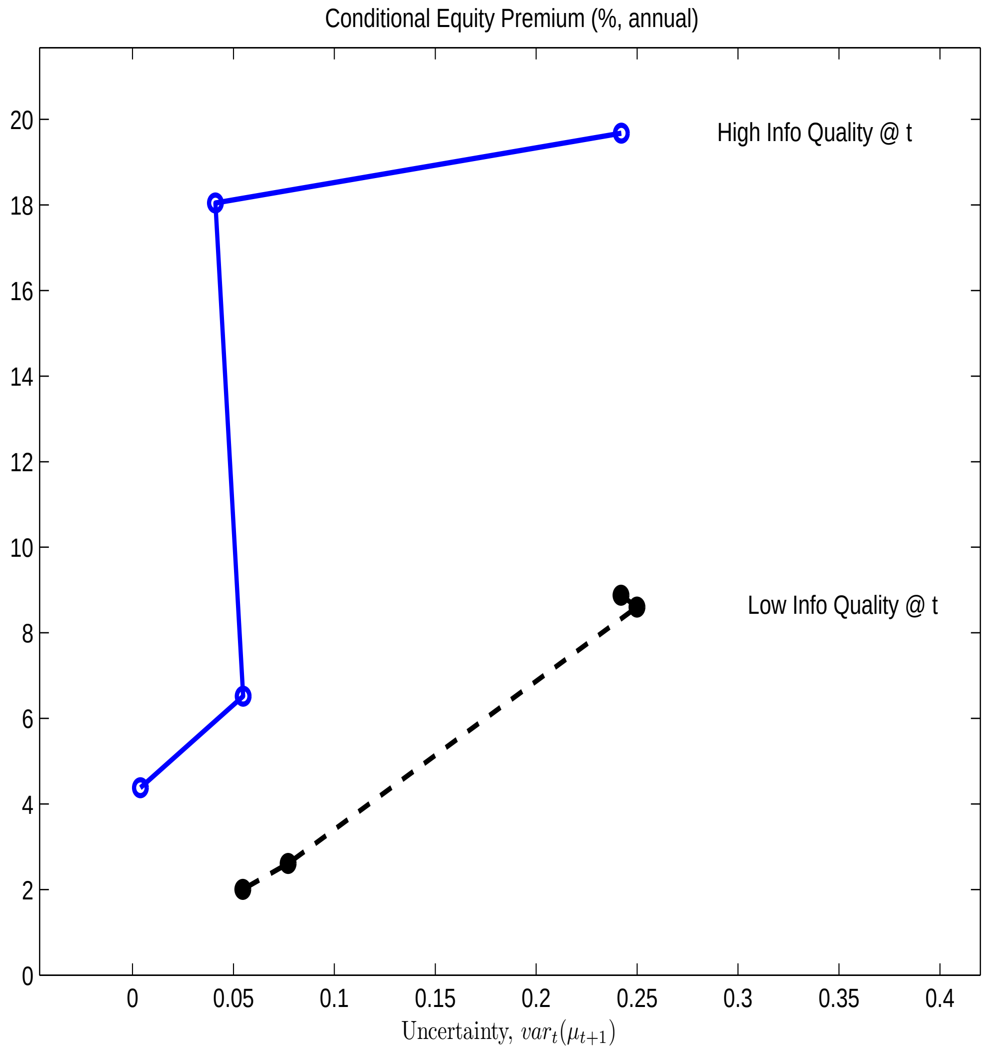

Figure 7: **Conditional Equity Premium versus Prior Uncertainty.** The vertical axis is annualized conditional equity premium with percentage unit. The horizontal axis is *prior uncertainty* which is the variance of μ_t in an agent's belief before she receives a signal y_t as defined in Section 2. The solid line shows the conditional equity premium at the high information quality state while the dashed line displays the conditional equity premium at the low information quality state. The same parameter values are used as Figure 1.

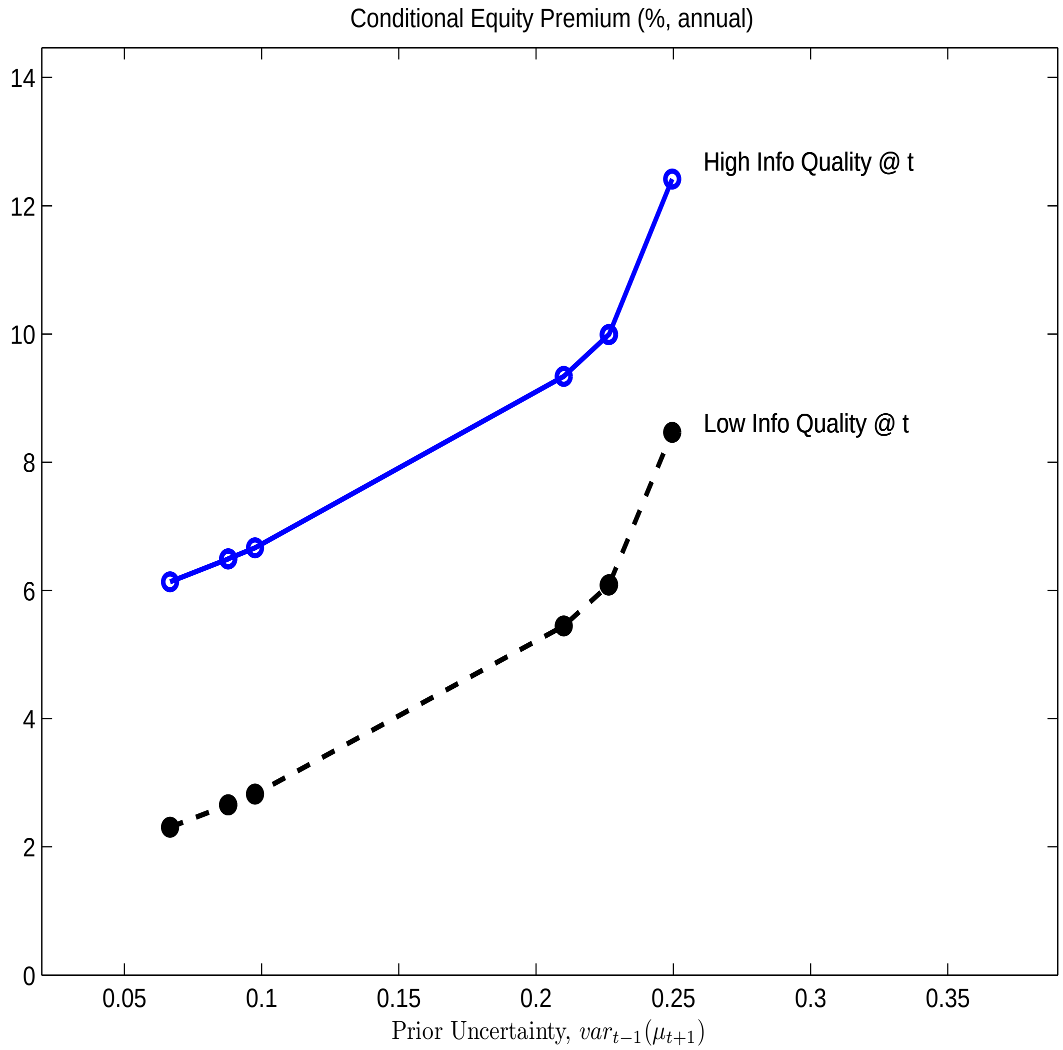

Figure 8: **Conditional Equity Premium versus log Dividend-Price Ratio.** The vertical axis is annualized conditional equity premium with percentage unit. The horizontal axis is log dividend price ratio. The solid line shows the conditional equity premium at the high information quality state while the dashed line displays the conditional equity premium at the low information quality state. The same parameter values are used as Figure 1.

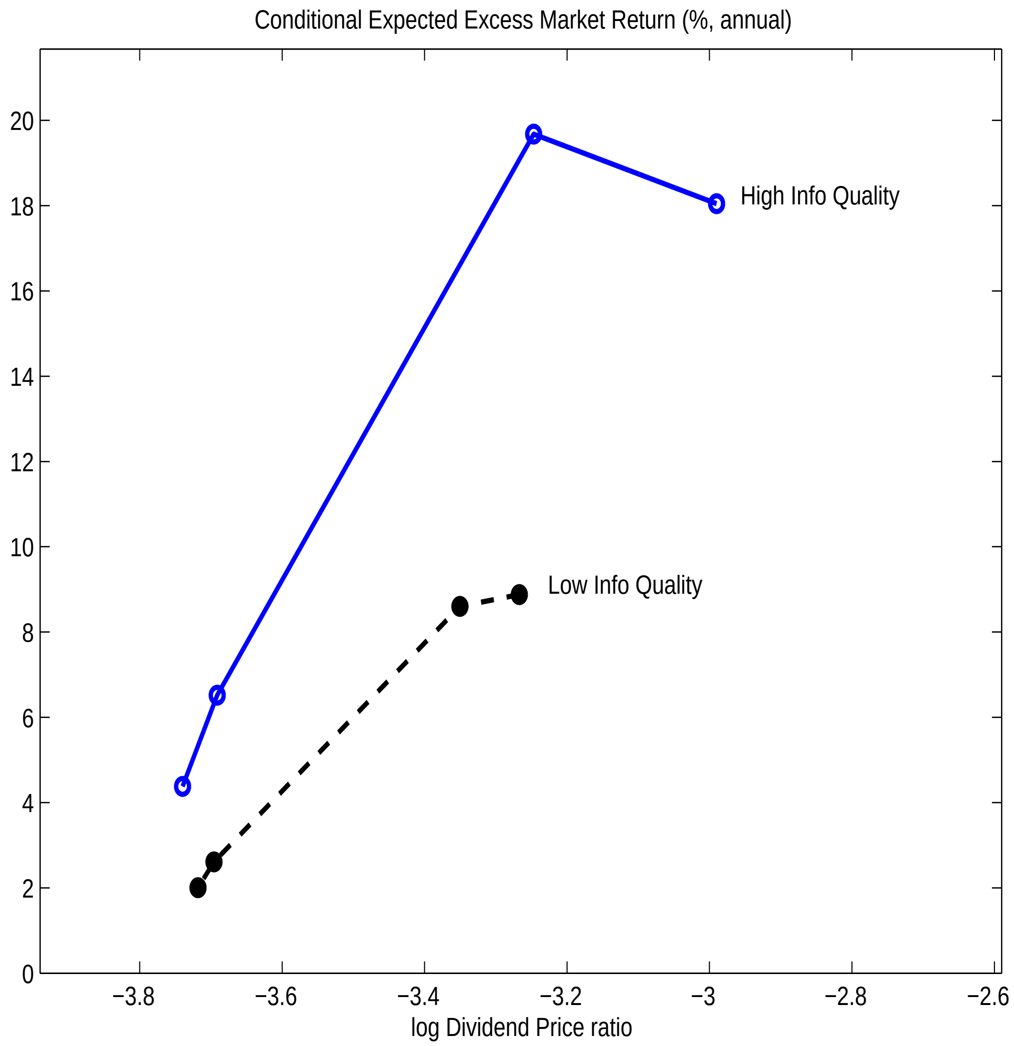

Figure 9: **Conditional Equity Premium versus log Dividend-Price Ratio.** The figures repeat Figure with different preferences. The figures in the top row show the equity premium by the preference with ambiguity aversion to signals ($\gamma = 2, \eta = 8.9$). The figures in the middle row display the equity premium by the preference with ambiguity aversion to mean growth ($\gamma = 2, \eta = 8.9$). The figures in the bottom row show the equity premium by the Epstein-Zin preference ($\gamma = \eta = 8.9$). The solid line shows the conditional equity premium at high Information Quality state (figures in the left columns) or high Prior Uncertainty state (figures in the right columns) while the dashed line displays the conditional equity premium at the low information quality state (figures in the left columns) or low uncertainty state (figures in the right columns). Prior Uncertainty is the variance of μ_t in an agent's belief before she receives a signal y_t as defined in Section 2. The same parameter values are used as Figure 1.

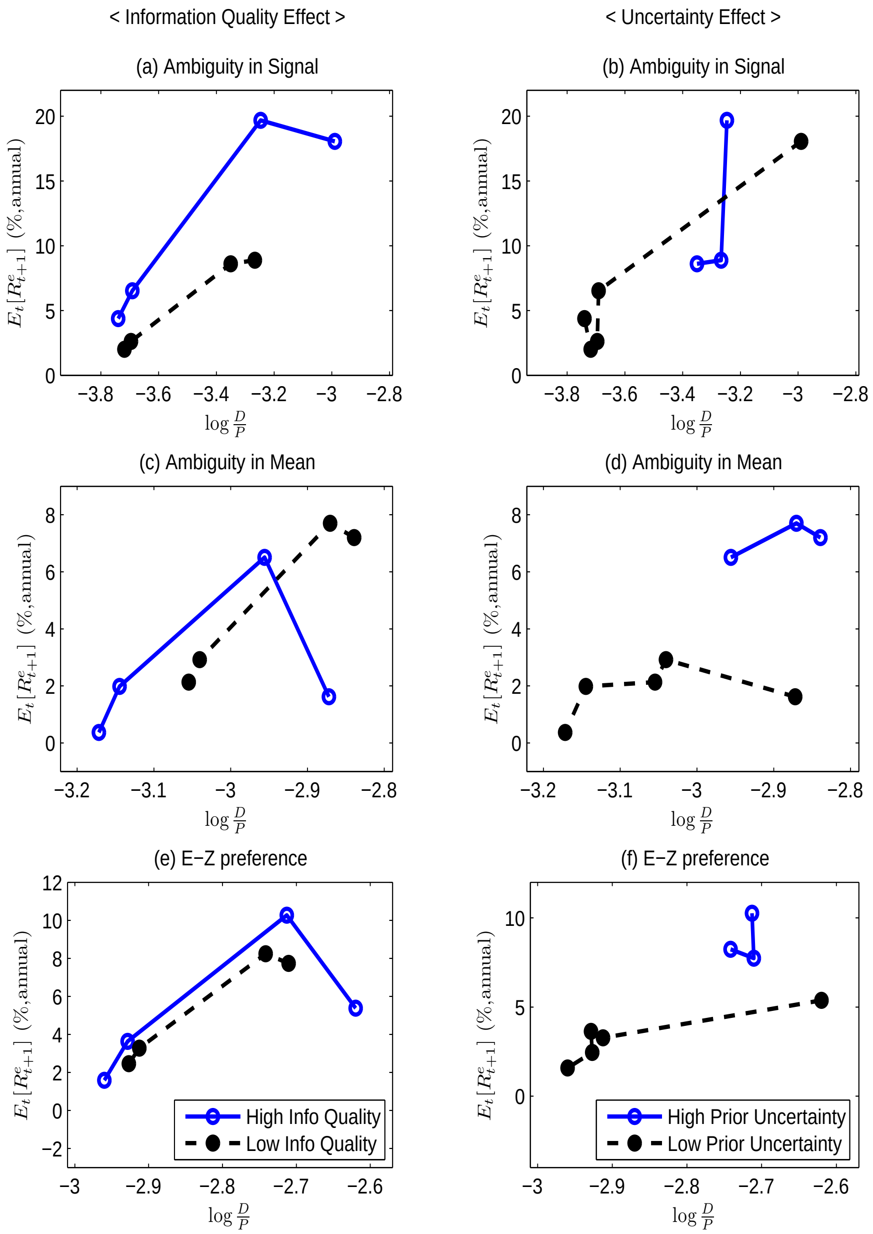

Figure 10: **Data from the Survey of Professional Forecasters.** The gray line in Figure (a) shows the median of the point forecasts by individual forecasters. The black line displays the historical real GDP growth. Figure (b) shows $\Phi^{-1}(Pr[\text{recession}])$ which is a nonlinear transformation of the median of the recession probability forecasts. $\Phi(\cdot)$ denotes a cumulative distribution function of a standard normal distribution. Figure (c) displays log of cross-sectional dispersion in point forecasts as a variance of the forecasts across the forecasters.

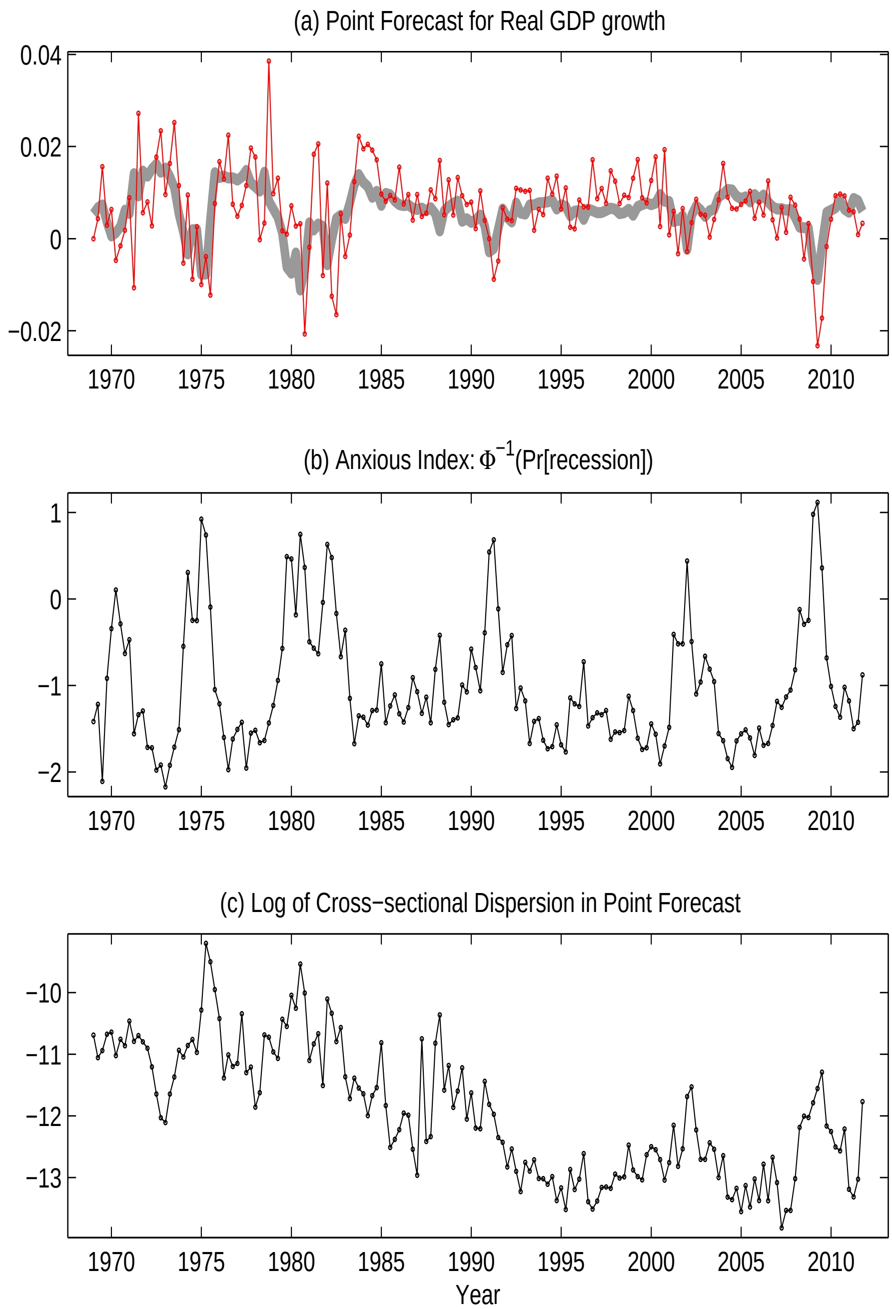

Figure 11: **Uncertainty Measures** The figure plots the different uncertainty measures defined in 4. Figure (a) Prior Market Uncertainty U_t, (b) Posterior Market Uncertainty U_t^*, (c) Prior Individual Uncertainty u_t, and (d) Conditional Variance of real GDP growth v_t. The black solid line displays the median of filtered distribution $p(\cdot|\mathcal{F}_t)$ while the gray area shows (25%,75%) quantile of the filtered distribution.

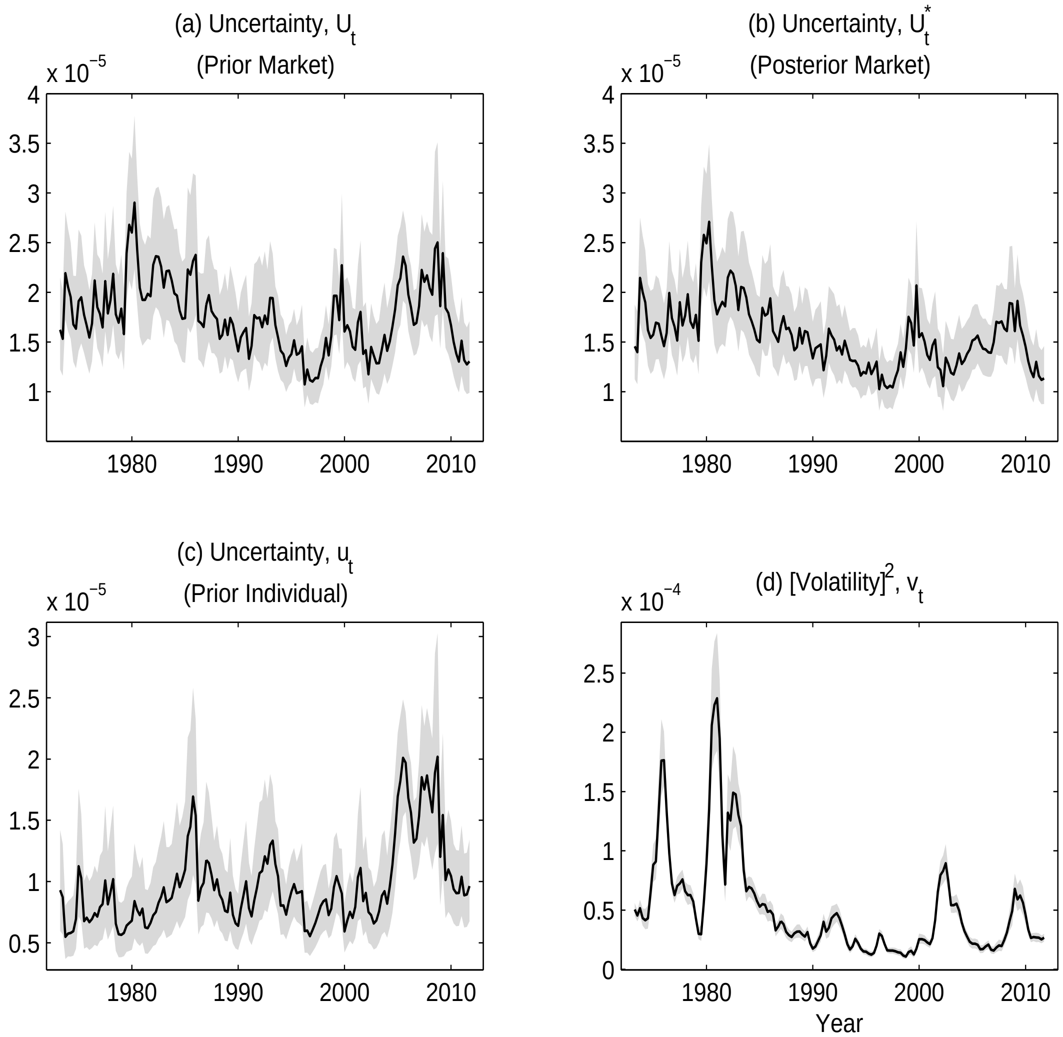

Figure 12: **Information Quality Measures and Implied Aggregate Signal** The figure plots the different information quality measures and the aggregate signal defined in 4. Figure (a) Information Quality (precision) q_t, (b) Implied Aggregate Signal y_t, (c) Prior Relative Uncertainty Q_t, and (d) Posterior Relative Uncertainty Q_t^*. The black solid line displays the median of filtered distribution $p(\cdot|\mathcal{F}_t)$ while the gray area shows (25%,75%) quantile of the filtered distribution.

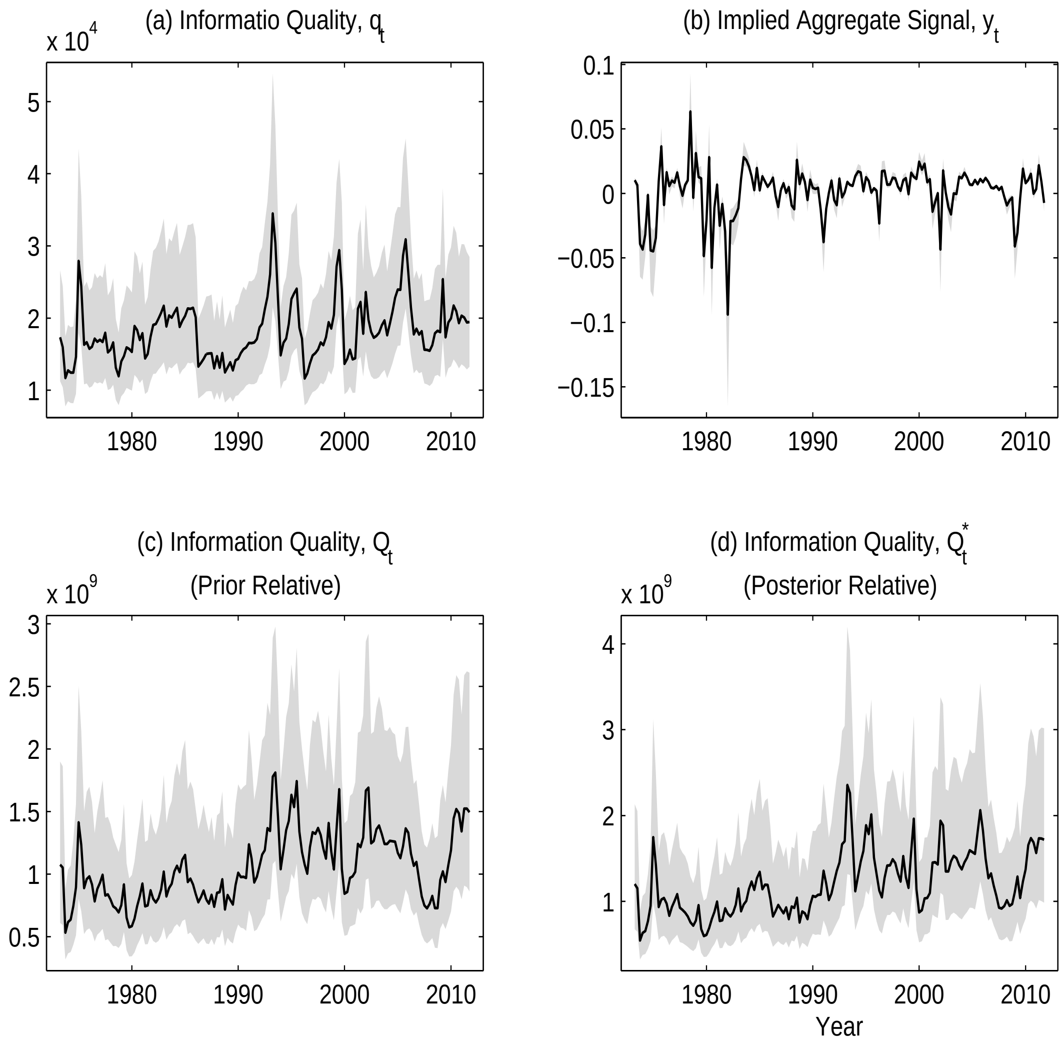

Figure 13: **Scatter Plots** The figure plots the scatter plots between several variables. Correlation of two variables and the t-statistic of the regression slope coefficient are reported at the top of each figures. In Figure (b) shows data with a 45-degree line. Each plots are explained in details in Section 6.

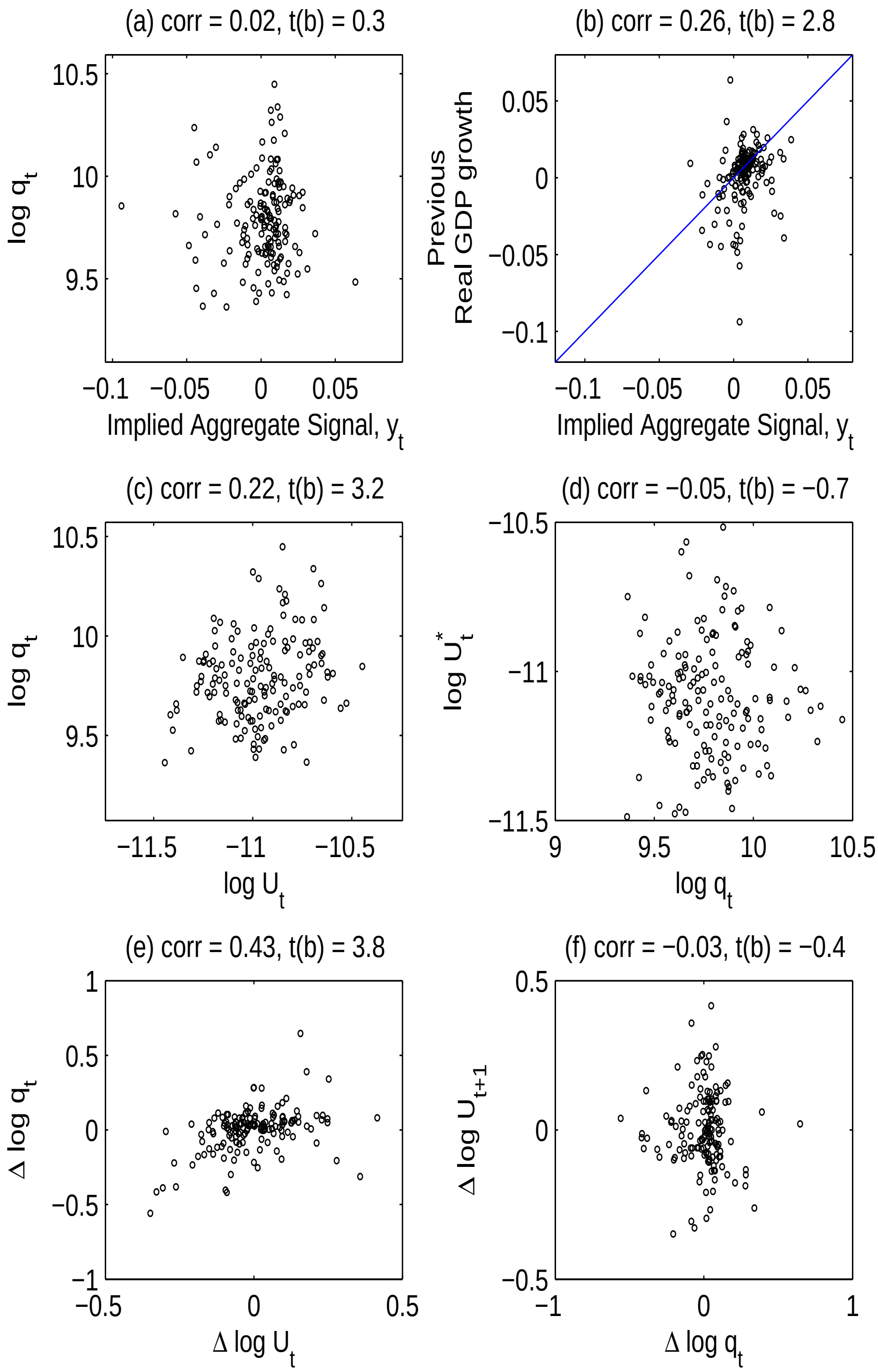

Figure 14: **Return-predictability of Information Quality at One-quarter Horizon.** Thick black solid line shows the forecasts from log of *information quality* $\log q_t$ in (a) or log of *prior relative information quality* $\log Q_t$ in (b). Thin red solid line shows quarterly excess market return. Both $\log q_t$ and $\log Q_t$ are the median of their filtered distribution.

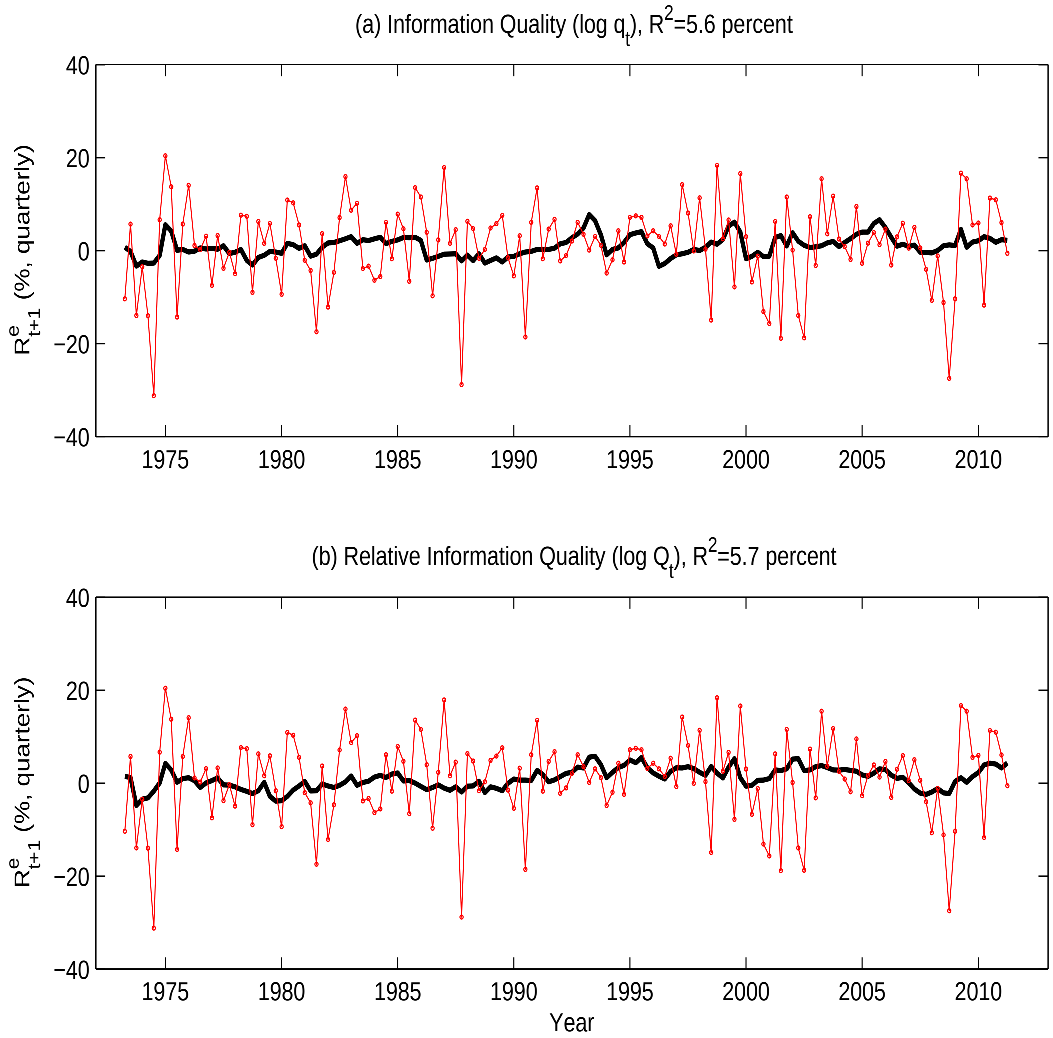

Figure 15: **Return-predictability of Information Quality and log D/P at Two-year Horizon.** Thick black solid line shows the forecasts from log of *prior relative information quality* $\log Q_t$ while thick gray line displays the forecasts from the log dividend price ratio. Thin red solid line shows two-year excess market return which is constructed from quarterly return.

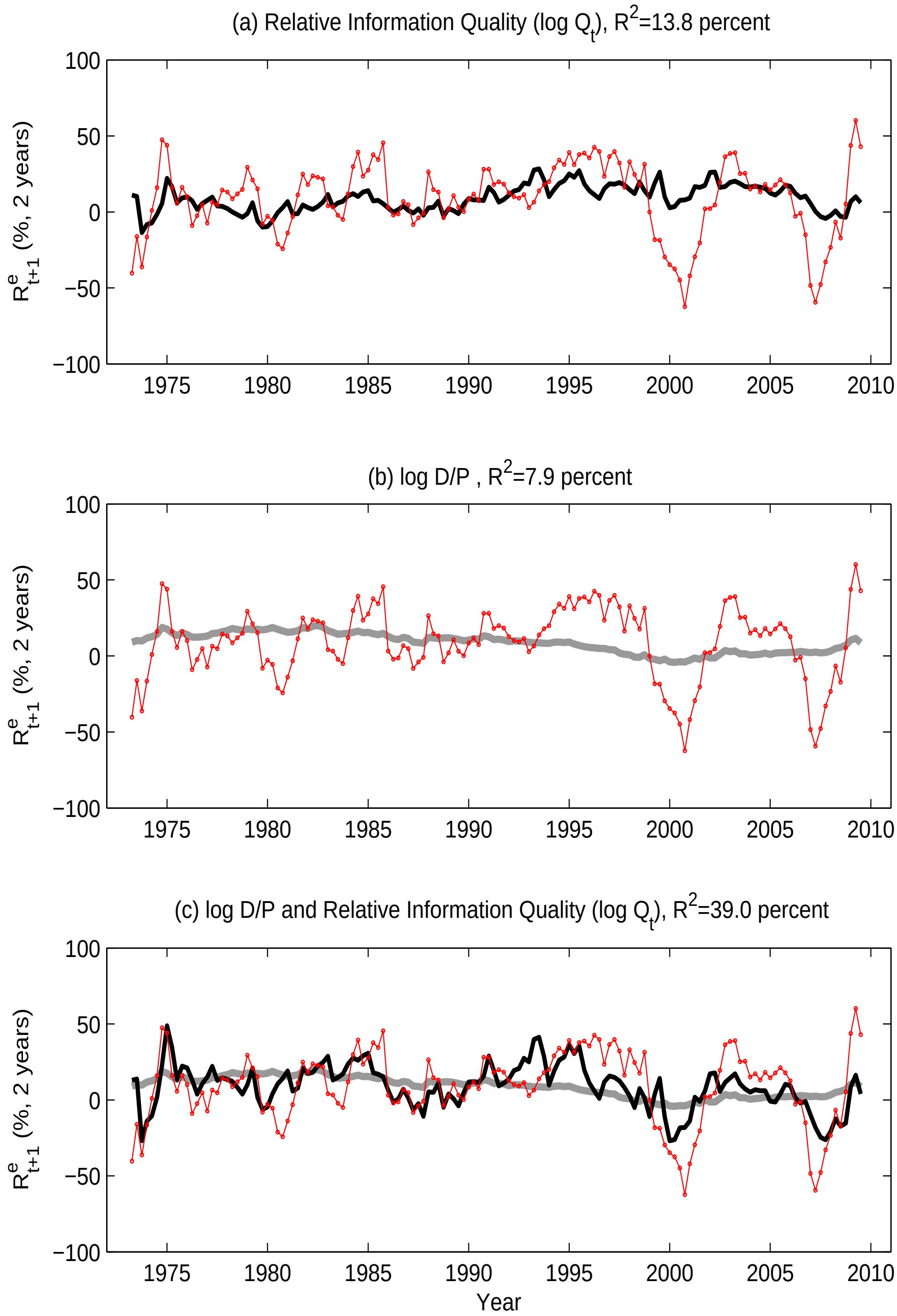

Figure 16: **Cross-sectional Variation and Pricing Errors.** This figure depicts average quarterly excess returns of 30 portfolios against model predicted excess returns. Test assets are 10 Size-, 10 Book-to-Market-, and 10 Momentum-sorted portfolios as used in Ozoguz (2009). *Info.Q*, *UNC*, and *News* denote $\Delta \log q_t$, $\Delta \log U_t$, and y_t, respectively as in Table 10. MktRf, SMB, and HML are Fama-French 3-factors and MOM is a momentum factor. In all plots, Size-, Book-to-Market-, and Momentum-sorted portfolios are expressed as a marker of circle(o), dot($\cdot$), and star(*), respectively. The diagonal line in all plots is a 45-degree line.

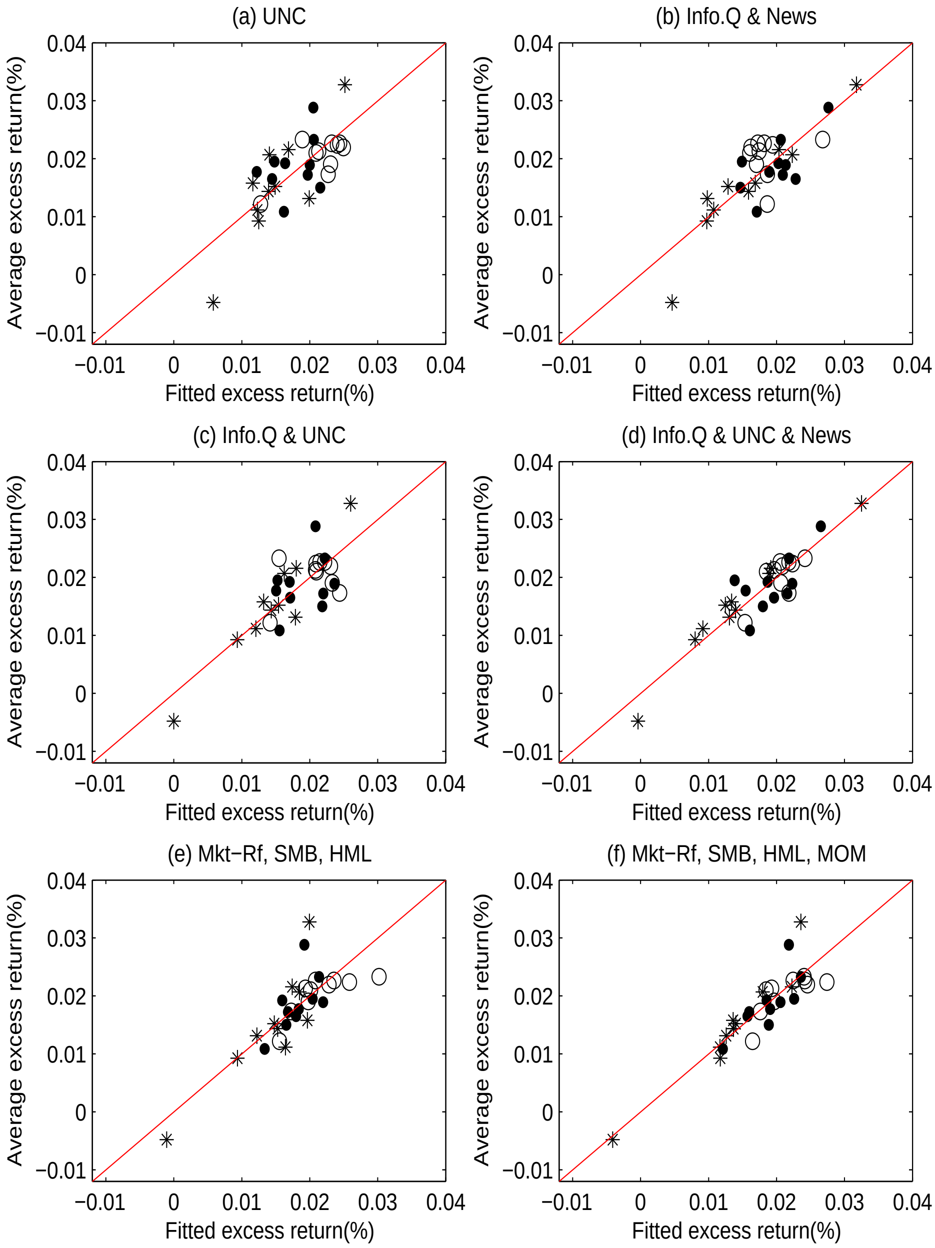

Table 1: **Model Implied Equity Premium and Other Moments.**

Panel A compares a preference proposed in Section 2 and a preference by Ju and Miao (2012) using two different parameter sets in Table 2. For each preference, three different signal structures are compared. The first column in each preference shows the case that an agent receives an uninformative signal $q_t = q = 0.5$ which is a pure noise. In the second column, an agent receives a signal with moderate level of quality $q_t = q = 0.75$. In the third column, the signal quality q_t changes over time. It switches between $q_t = q_A = 0.95$ and $q_t = q_I = 0.55$. Panel B repeats Panel A with a CRRA utility and a CRRA with ambiguity aversion. $E[R^e]$ is the unconditional equity premium which is the unconditional expected market return less the risk free rate. $E[R^e|q_A]$ and $E[R^e|q_I]$ are the conditional equity premium when information quality is high $q_t = q_A$ and $q_t = q_I$, respectively. $Vol(R^e)$ is the volatility of the excess market return. $E[R_f]$ is the unconditional mean of the risk free rate. $E[\log \frac{D}{P}]$ is the unconditional mean of the log dividend-price ratio. All numbers are expressed as annualized percentage except for the log dividend price ratio. Ambiguity parameter $\eta = 8.9$ and EIS $\rho^{-1} = 1.5$ are reported in the table. The subjective discount rate is set as $\beta = 0.973$.

Panel A: Different Types of Ambiguity Aversion

		Parameter Set I			Parameter Set II					
Preference (γ, η, ρ^{-1})		Ambiguity in Mean (2.0, 8.9, 1.5)			Ambiguity in Mean (2.0, 8.9, 1.5)			Ambiguity in Signal (2.0, 8.9, 1.5)		
q_t =Pr[correct signal]	q_I	.50	.75	.55	.50	.75	.55	.50	.75	.55
	q_A	.50	.75	.95	.50	.75	.95	.50	.75	.95
$E[R^e]$		8.9	6.6	6.1	3.2	2.6	2.2	2.3	4.0	5.1
$E[R^e\|q_t = q_I]$		8.9	6.6	9.0	3.2	2.6	3.1	2.3	4.0	3.1
$E[R^e\|q_t = q_A]$		8.9	6.6	3.2	3.2	2.6	1.2	2.3	4.0	7.0
$Vol(R^e)$		17.4	17.1	19.8	13.7	13.3	13.5	18.8	20.0	21.0
$E[R_f]$		2.2	2.8	2.8	3.7	3.8	3.8	3.4	-0.1	0.4
$E[\log \frac{D}{P}]$		2.6	2.8	3.0	2.9	3.0	3.1	3.4	4.4	3.6

Panel B: Decomposition of Equity Premium (Parameter Set II)

Preference		CRRA			Ambiguity in Signal					
(γ, η, ρ^{-1})		(2.0, 2.0, 0.5)			(2.0, 8.9, 0.5)			(2.0, 8.9, 1.5)		
q_t =Pr[correct signal]	q_I	.50	.75	.55	.50	.75	.55	.50	.75	.55
	q_A	.50	.75	.95	.50	.75	.95	.50	.75	.95
$E[R^e]$		0.3	0.2	0.2	1.8	3.0	3.7	2.3	4.0	5.1
$E[R^e\|q_t = q_I]$		0.3	0.2	0.3	1.8	3.0	2.4	2.3	4.0	3.1
$E[R^e\|q_t = q_A]$		0.3	0.2	0.1	1.8	3.0	5.0	2.3	4.0	7.0
$Vol(R^e)$		11.4	11.1	10.8	17.3	17.9	18.4	18.8	20.0	21.0
$E[R_f]$		6.7	6.7	6.8	6.2	2.9	3.6	3.4	-0.1	0.4
$E[\log \frac{D}{P}]$		2.8	2.8	2.8	2.8	3.3	3.0	3.4	4.4	3.6

Table 2: **Parameters for a Markov Switching Model.**

The table reports the parameter values in Section 2. Parameter set I is from Ju and Miao (2012).

Parameters	$\mu_H(\%)$	$\mu_L(\%)$	$\sigma_c(\%)$	p_{ll}	p_{hh}	p_{ii}	p_{aa}	ϕ_μ	ϕ_e	$g_d(\%)$	$\sigma_d(\%)$
Set I (disaster)	2.25	-6.79	3.13	0.52	0.98	0.90	0.90	2.74	2.74	-3.23	8.40
Set II (recession)	2.50	-1.90	1.30	0.54	0.93	0.90	0.90	4.50	1.50	-8.23	7.00

Table 3: **Sample correlation**

Panel A reports the correlation between variables estimated or taken from the SPF. Panel B displays the correlation between each variable in Panel A and the predictors from Goyal and Welch (2008)

Panel A: Correlation between variables from the SPF

	q_t	Q_t	v_t	u_t	U_t	U_t^*	y_t	$\hat{\mu}_t^*$	Disp.
Information Quality	1.00	0.65	-0.06	0.52	0.26	0.00	0.02	0.01	-0.10
Prior Relative Information Quality	0.65	1.00	-0.27	0.04	-0.54	-0.69	0.14	0.13	-0.31
Volatility^2	-0.06	-0.27	1.00	-0.26	0.30	0.45	-0.29	-0.23	0.63
Prior Individual Uncertainty	0.52	0.04	-0.26	1.00	0.49	0.12	0.06	0.07	-0.27
Prior Market Uncertainty	0.26	-0.54	0.30	0.49	1.00	0.92	-0.20	-0.23	0.30
Posterior Market Uncertainty	0.00	-0.69	0.45	0.12	0.92	1.00	-0.26	-0.29	0.45
Implied Aggregate Signal	0.02	0.14	-0.29	0.06	-0.20	-0.26	1.00	0.72	-0.35
Posterior Forecast	0.01	0.13	-0.23	0.07	-0.23	-0.29	0.72	1.00	-0.44
Dispersion (variance)	-0.10	-0.31	0.63	-0.27	0.30	0.45	-0.35	-0.44	1.00

Panel B: Correlation between each variable in Panel A and the predictors from Goyal and Welch (2008)

	q_t	Q_t	v_t	u_t	U_t	U_t^*	y_t	$\hat{\mu}_t^*$	Disp.
cay	0.00	0.27	-0.14	-0.28	-0.34	-0.27	0.03	-0.09	-0.12
Dividend price ratio	-0.20	-0.46	0.56	-0.24	0.36	0.53	-0.25	-0.19	0.55
Dividend payout ratio	0.09	0.02	0.12	0.05	0.06	0.05	-0.17	-0.28	-0.02
Dividend yield	-0.18	-0.44	0.56	-0.25	0.35	0.52	-0.22	-0.19	0.56
Earning price ratio	-0.23	-0.42	0.41	-0.25	0.28	0.43	-0.10	0.03	0.49
Stock volatility	-0.01	-0.04	0.03	0.08	0.04	0.02	-0.26	-0.31	0.01
Book-to-market	-0.16	-0.49	0.67	-0.29	0.44	0.63	-0.26	-0.16	0.66
Net equity expansion	0.12	0.34	0.15	-0.45	-0.30	-0.19	0.09	0.21	0.09
T-bill rate	-0.29	-0.50	0.45	-0.36	0.32	0.53	-0.14	-0.17	0.39
Long term yield	-0.17	-0.37	0.53	-0.35	0.27	0.47	-0.13	-0.09	0.41
Term Spread	0.32	0.44	-0.07	0.19	-0.23	-0.34	0.08	0.20	-0.15
Long term return	0.09	0.09	0.06	0.05	-0.04	-0.06	-0.19	-0.04	0.05
Default yield spread	0.06	-0.27	0.70	0.03	0.41	0.46	-0.42	-0.41	0.47
Default return spread	0.02	0.07	0.08	-0.13	-0.10	-0.06	0.03	0.01	0.07
Inflation	-0.20	-0.41	0.32	-0.22	0.33	0.46	-0.09	-0.15	0.44
Inv/Capital ratio	-0.19	-0.20	-0.10	-0.24	0.07	0.17	-0.00	-0.05	0.04

Table 4: **Excess Market Return Predictability of the Variables from the SPF**

This table compares the prediction power of the variables estimated or taken from the *Survey of Professional Forecasters* (SPF). The market return is the quarterly value-weighted CRSP index (NYSE/AMEX/NASDAQ) less the 90 day T-Bill rate (in percentages). Panel A shows return predictability of Individual Uncertainty u_t, Weight in belief-updating $q_t \times u_t$, Information Quality q_t, Prior Relative Information Quality Q_t, and Posterior Relative Information Quality Q_t^* as defined in Section 4. Panel B repeats Panel A with two-quarter horizon. Panel C shows return predictability of variance of real GDP growth u_t, Prior Forecast $\hat{\mu}_t$, Posterior Forecast $\hat{\mu}_t^*$, Prior Market Uncertainty U_t, and Posterior Market Uncertainty U_t^* as defined in Section 4. Panel D repeats Panel C with two-quarter horizon. All the regressors are scaled by their sample standard deviation so that the regression coefficients measure the average effect of one standard deviation increase in a predictor on to an increase in the percentage excess market returns in next quarter. Next to the regression coefficients, t-statistics are reported in parentheses and it is based on Newey-West adjusted standard errors with lag length equal to the number of overlapping quarters in forecasting horizon. The full sample period of Survey of Professional Forecasters is from 1968:Q4 until 2011:Q3. However, I have removed the first 16 quarter observations to mitigate the effect of the priors. The data used in predictive regressions are from 1973:Q1 to 2010:Q4.

Panel A: Horizon=1Q										
Predictors	u_t		$q_t \times u_t$		q_t		Q_t		Q_t^*	
Slope	0.3	(0.4)	1.1	(1.5)	2.0	(2.7)	2.0	(2.5)	1.9	(2.6)
Intercept	0.0	(0.0)	-1.1	(-0.7)	-8.0	(-2.3)	-6.9	(-2.1)	-5.8	(-2.1)
$R^2(\%)$	0.1		1.3		4.8		4.6		4.3	
Panel B: Horizon=2Q										
Predictors	u_t		$q_t \times u_t$		q_t		Q_t		Q_t^*	
Slope	-0.6	(-0.2)	1.7	(0.8)	4.5	(2.1)	5.4	(2.0)	5.0	(2.0)
Intercept	5.5	(0.6)	0.6	(0.1)	-15.6	(-1.5)	-18.0	(-1.6)	-13.6	(-1.4)
$R^2(\%)$	0.1		0.8		5.6		8.4		6.9	
Panel C: Horizon=1Q										
Predictors	v_t		$\hat{\mu}_t$		$\hat{\mu}_t^*$		U_t		U_t^*	
Slope	0.3	(0.5)	-0.3	(-0.4)	0.3	(0.4)	-0.4	(-0.4)	-0.6	(-0.8)
Intercept	0.5	(0.5)	1.3	(1.0)	0.7	(0.7)	2.7	(0.7)	4.2	(1.1)
$R^2(\%)$	0.1		0.1		0.1		0.2		0.5	
Panel D: Horizon=2Q										
Predictors	v_t		$\hat{\mu}_t$		$\hat{\mu}_t^*$		U_t		U_t^*	
Slope	-0.1	(-0.1)	-1.8	(-1.0)	1.2	(0.8)	-1.9	(-0.8)	-2.3	(-0.9)
Intercept	3.9	(1.0)	5.8	(1.6)	2.9	(1.0)	13.5	(1.1)	15.9	(1.2)
$R^2(\%)$	0.0		0.9		0.4		1.1		1.5	

Table 5: **Excess Market Return Predictability: One-Quarter Horizon.**

The table reports predictive regressions of the market excess return over one-quarter horizon on lagged predictor variables. The market return is the quarterly value-weighted CRSP index (NYSE/AMEX/NASDAQ) less the 90 day T-Bill rate (in percentages). The columns from (1) to (4) show each multiple regression result with four different forms of information quality measure constructed from information up to time $t-1$ as explained in Section 4: (1) the precision q_t, (2) $\log q_t$, (3) $-q_t^{-1}$, and (4) $-q_t^{-2}$. Panel A reports univariate predictive regressions on information quality sequentially estimated from the SPF. Panel B repeats Panel A with additional predictors: the log dividend-price ratio and *cay*. Panel C repeats Panel A with additional predictors: fifteen predictors considered in Goyal and Welch (2008). They are available up to 2010:Q4 from Ivo Welchs website. Predictive regressions in Panel B and C are multiple regressions with all variables listed. All the regressors are scaled by their sample standard deviation so that the regression coefficients measure the average effect of one standard deviation increase in a predictor on to an increase in the percentage excess market returns in next quarter. Next to the regression coefficients, t-statistics are reported in parentheses. The full sample period of Survey of Professional Forecasters is from 1968:Q4 until 2011:Q3. However, I have removed the first 16 quarter observations to mitigate the effect of the priors. The data used in predictive regressions are from 1973:Q1 to 2010:Q4.

Panel A: univarite regression			(1)	q_t	(2)	$\log q_t$	(3)	$-q_t^{-1}$	(4)	$-q_t^{-2}$
Information quality			2.0	(2.7)	2.2	(2.9)	2.3	(3.0)	2.4	(3.0)
Intercept			-8.0	(-2.3)	-99.1	(-2.9)	12.1	(3.2)	6.8	(3.4)
$R^2(\%)$			4.8		5.6		6.2		6.5	
Panel B: with $\log \frac{D}{P}$ *and cay*			(1)	q_t	(2)	$\log q_t$	(3)	$-q_t^{-1}$	(4)	$-q_t^{-2}$
Information quality			2.2	(3.0)	2.4	(3.2)	2.5	(3.4)	2.5	(3.4)
cay	1.6	(2.5)	1.5	(2.4)	1.5	(2.4)	1.5	(2.5)	1.5	(2.5)
log D/P	0.5	(0.6)	0.9	(1.2)	0.9	(1.2)	0.9	(1.2)	0.9	(1.2)
Intercept	4.3	(0.7)	-1.9	(-0.3)	-100.1	(-3.0)	20.0	(2.8)	14.1	(2.2)
$R^2(\%)$	3.5		8.8		9.7		10.3		10.7	
Panel C: with all 15 predictors			(1)	q_t	(2)	$\log q_t$	(3)	$-q_t^{-1}$	(4)	$-q_t^{-2}$
Information quality			2.0	(2.9)	2.2	(3.1)	2.4	(3.2)	2.5	(3.2)
cay	4.9	(3.6)	4.5	(3.5)	4.4	(3.5)	4.4	(3.5)	4.3	(3.4)
Dividend price ratio	0.7	(0.1)	2.5	(0.4)	2.6	(0.4)	2.5	(0.4)	2.4	(0.4)
Dividend payout ratio	-1.7	(-1.4)	-1.9	(-1.5)	-1.9	(-1.5)	-1.8	(-1.5)	-1.8	(-1.5)
Dividend yield	-3.3	(-0.6)	-3.2	(-0.6)	-2.9	(-0.6)	-2.5	(-0.5)	-2.2	(-0.4)
Stock volatility	-1.5	(-0.9)	-1.2	(-0.7)	-1.1	(-0.6)	-1.0	(-0.6)	-1.0	(-0.6)
Book-to-market	5.1	(1.4)	3.7	(1.1)	3.4	(1.0)	3.2	(0.9)	3.0	(0.9)
Net equity expansion	-1.0	(-1.2)	-1.2	(-1.5)	-1.2	(-1.5)	-1.2	(-1.4)	-1.2	(-1.4)
Long term yield	-4.6	(-2.6)	-4.4	(-2.6)	-4.4	(-2.6)	-4.5	(-2.6)	-4.6	(-2.7)
Term Spread	0.1	(0.1)	-0.3	(-0.3)	-0.4	(-0.4)	-0.4	(-0.4)	-0.4	(-0.4)
Long term return	1.3	(1.2)	1.2	(1.1)	1.2	(1.1)	1.2	(1.1)	1.1	(1.1)
Default yield spread	1.9	(1.2)	1.5	(0.9)	1.3	(0.8)	1.2	(0.8)	1.1	(0.7)
Default return spread	2.9	(2.2)	3.0	(2.4)	3.0	(2.3)	3.0	(2.3)	2.9	(2.3)
Inflation	0.4	(0.5)	0.5	(0.5)	0.6	(0.6)	0.6	(0.7)	0.7	(0.7)
Inv/Capital ratio	-0.6	(-0.4)	-0.3	(-0.2)	-0.2	(-0.2)	-0.1	(-0.1)	-0.1	(-0.1)
Intercept	-16.7	(-0.5)	-10.5	(-0.3)	-99.7	(-2.5)	16.0	(0.5)	12.2	(0.4)
$R^2(\%)$	17.7		21.4		22.1		22.6		22.9	

Table 6: **Excess Market Return Predictability: Control of Uncertainty.**

The table repeats the analysis of Table 5 with Relative Information Quality Q_t (defined in Section 4) so that the effect of uncertainty in the predictive regression is controlled. Panel D reports multiple regressions with all variables listed in Table 5C. The coefficients and t-statistics of fifteen predictors are omitted in the table. Forecasting horizon is one quarter and the sample period is the same as in Table 5.

Panel A: univarite regression			(1)	Q_t	(2)	$\log Q_t$	(3)	$-Q_t^{-1}$	(4)	$-Q_t^{-2}$
Relative Info-quality			2.0	(2.5)	2.2	(2.8)	2.4	(3.1)	2.5	(3.4)
Intercept			-6.9	(-2.1)	-182.7	(-2.8)	10.4	(3.5)	5.8	(3.9)
$R^2(\%)$			4.6		5.7		6.7		7.4	
Panel B: with $\log \frac{D}{P}$			(1)	Q_t	(2)	$\log Q_t$	(3)	$-Q_t^{-1}$	(4)	$-Q_t^{-2}$
Relative Info-quality			3.0	(3.6)	3.3	(4.1)	3.4	(4.5)	3.5	(4.8)
log D/P	0.8	(1.0)	2.1	(2.7)	2.3	(2.9)	2.3	(3.0)	2.2	(2.9)
Intercept	7.1	(1.2)	5.9	(1.0)	-251.8	(-3.9)	32.6	(4.3)	25.0	(3.8)
$R^2(\%)$	0.7		8.7		10.5		11.7		12.2	
Panel C: with $\log \frac{D}{P}$ *and cay*			(1)	Q_t	(2)	$\log Q_t$	(3)	$-Q_t^{-1}$	(4)	$-Q_t^{-2}$
Relative Info-quality			2.7	(2.7)	3.1	(3.2)	3.4	(3.5)	3.4	(3.8)
cay	1.6	(2.5)	0.5	(0.6)	0.3	(0.4)	0.2	(0.2)	0.2	(0.3)
log D/P	0.5	(0.6)	1.9	(2.0)	2.2	(2.3)	2.2	(2.4)	2.2	(2.4)
Intercept	4.3	(0.7)	5.1	(0.8)	-240.8	(-3.1)	31.6	(3.2)	24.2	(3.0)
$R^2(\%)$	3.5		8.9		10.6		11.8		12.2	
Panel D: with all 15 predictors			(1)	Q_t	(2)	$\log Q_t$	(3)	$-Q_t^{-1}$	(4)	$-Q_t^{-2}$
Relative Info-quality			2.3	(2.3)	2.9	(2.9)	3.2	(3.3)	3.3	(3.6)
$R^2(\%)$	17.7		20.1		21.3		22.3		22.7	

Table 7: **Excess Market Return Predictability: Longer Horizon Result.**

The table reports predictive regressions of the market excess return on lagged predictor variables. Forecasting horizon is from one to twelve quarters. Panels A and B show results of predictive regressions by Information Quality q_t and Relative Information Quality Q_t (defined in Section 4), respectively. Panel D repeats Panel A with the log dividend-price ratio as an additional predictor. Panel E reports multiple regressions with Q_t and all variables listed in Table 5C. The coefficients and t-statistics of fifteen predictors are omitted in the table. Next to the regression coefficients, t-statistics are reported in parentheses and it is based on Newey-West adjusted standard errors with lag length equal to the number of overlapping quarters in forecasting horizon. The data used in predictive regressions are from 1973:Q1 to 2011:Q2.

Panel A: univarite regression of information quality										
Horizon (quarter)	H=1		H=2		H=4		H=8		H=12	
Info-quality q_t	2.0	(2.7)	3.4	(2.9)	4.5	(2.1)	2.8	(1.3)	1.5	(0.5)
R^2(%)	4.8		6.4		5.6		1.5		0.3	
Panel B: univarite regression of information quality controlled by uncertainty										
Horizon (quarter)	H=1		H=2		H=4		H=8		H=12	
Relative Info-quality Q_t	2.0	(2.5)	3.7	(2.4)	5.4	(2.0)	8.3	(3.5)	9.2	(3.4)
R^2(%)	4.6		7.2		8.4		13.2		12.5	
Panel C: $\log \frac{D}{P}$ *only*										
Horizon (quarter)	H=1		H=2		H=4		H=8		H=12	
log D/P	0.8	(1.0)	1.8	(1.3)	3.7	(1.5)	6.4	(1.5)	8.6	(1.7)
R^2(%)	0.7		1.7		3.8		8.0		11.1	
Panel D: with $\log \frac{D}{P}$										
Horizon (quarter)	H=1		H=2		H=4		H=8		H=12	
Relative Info-quality Q_t	3.0	(3.6)	5.6	(3.6)	8.8	(3.3)	13.9	(5.3)	16.8	(7.4)
log D/P	2.1	(2.7)	4.3	(3.2)	7.6	(3.3)	12.7	(3.6)	16.4	(4.5)
R^2(%)	8.7		15.3		21.5		37.9		44.1	
Panel E: with all 15 predictors										
Horizon (quarter)	H=1		H=2		H=4		H=8		H=12	
Relative Info-quality Q_t	2.3	(2.3)	4.3	(2.8)	5.6	(2.4)	9.8	(3.3)	9.0	(3.7)
R^2(%)	20.1		26.8		36.9		54.1		64.8	

Table 8: **Excess Market Return Predictability: Sub-periods Result.**

The table repeats the analysis of Table 6 for two sub-periods. The sub-period in Panel A covers from 1973:Q1 to 1991:Q1 while Panel B from 1991:Q1 to 2011:Q2. Forecasting horizon is one quarter as in Table 6.

Panel A: 1973:Q1∼1991:Q1			(1)	Q_t	(2)	$\log Q_t$	(3)	$-Q_t^{-1}$	(4)	$-Q_t^{-2}$
Relative Info-quality Q_t			3.3	(3.2)	3.2	(3.0)	3.1	(3.0)	3.0	(3.2)
$R^2(\%)$			11.0		10.5		10.0		9.4	
Relative Info-quality Q_t			2.6	(2.6)	2.5	(2.5)	2.4	(2.4)	2.3	(2.3)
cay	3.0	(2.8)	2.2	(2.1)	2.1	(2.0)	2.1	(2.0)	2.1	(1.9)
log D/P	2.8	(2.4)	2.6	(2.4)	2.7	(2.4)	2.7	(2.4)	2.7	(2.4)
$R^2(\%)$	14.9		21.2		20.6		20.0		19.4	
with all 15 predictors										
Relative Info-quality Q_t			2.1	(2.2)	2.2	(2.2)	2.2	(2.1)	2.2	(2.0)
$R^2(\%)$	40.8		43.8		43.8		43.7		43.5	
Panel B: 1991:Q1∼2011:Q2			(1)	Q_t	(2)	$\log Q_t$	(3)	$-Q_t^{-1}$	(4)	$-Q_t^{-2}$
Relative Info-quality Q_t			1.6	(1.5)	2.0	(1.8)	2.3	(2.1)	2.5	(2.3)
$R^2(\%)$			3.5		5.1		6.8		8.3	
Relative Info-quality Q_t			1.7	(1.4)	2.2	(1.7)	2.6	(2.0)	2.9	(2.3)
cay	0.4	(0.4)	-0.4	(-0.4)	-0.7	(-0.6)	-1.0	(-0.8)	-1.1	(-1.0)
log D/P	1.3	(0.9)	1.6	(1.2)	1.8	(1.3)	1.9	(1.4)	2.0	(1.5)
$R^2(\%)$	3.2		6.3		8.1		10.1		12.0	
with all 15 predictors										
Relative Info-quality Q_t			1.0	(0.9)	1.5	(1.1)	2.0	(1.4)	2.4	(1.6)
$R^2(\%)$	21.6		22.3		22.8		23.5		24.3	

Table 9: **Excess Market Return Predictability: Out-of-sample Test.**

The table reports out-of-sample R^2 (percentages). The results for three different prediction periods are included: (1) 1980:Q3∼2010:Qt, (2) 1985:Q3∼ 2010:Q4, and (3) 1990:Q3∼2010:Q4. Out-of-sample R^2 is calculated as $R^2 = (1 - var(\hat{e}_t)/var(R_t^e)) \times 100$ where $\hat{e}_t$ is a forecasting error and R_t^e is the excess market return in the prediction period. The regression coefficients are estimated at every quarter t using the data from 1973:Q1 up to t to predict R_{t+1}^e. Panel B shows bivariate predictive regressions with each predictor variable from Goyal and Welch (2008) and Relative Information Quality $-1/Q_t^{-2}$.

Prediction period	(1) *1980:Q3∼2010:Q4*			(2) *1985:Q3∼2010:Q4*			(3) *1990:Q3∼2010:Q4*		
#Horizon (quarter)	1	2	4	1	2	4	1	2	4
Panel A: Univariate regression									
Relative Info-quality									
Q_t	-3.1	1.4	5.7	-4.5	-0.8	6.0	-1.1	4.0	9.0
$\log Q_t$	-0.3	4.6	8.9	-1.4	3.1	9.9	2.3	8.5	13.3
$-Q_t^{-1}$	2.0	7.1	11.1	1.1	6.2	12.6	4.7	11.6	16.1
$-Q_t^{-2}$	3.5	8.5	12.1	2.9	8.2	13.9	6.1	13.2	17.2
cay	-7.9	-7.9	-3.5	-3.5	-1.0	4.3	-0.6	4.8	12.7
Dividend price ratio	-6.4	-7.1	-7.2	-5.0	-4.8	-7.7	-6.4	-7.9	-8.1
Dividend payout ratio	-6.6	-5.1	-2.4	-6.7	-4.7	0.6	-5.9	-2.8	1.6
Dividend yield	-6.0	-5.4	-7.1	-5.4	-5.7	-7.6	-6.9	-7.4	-6.2
Earning price ratio	-7.6	-9.0	-9.9	-6.7	-7.6	-11.3	-6.1	-8.7	-9.9
Stock volatility	-24.6	-27.4	-8.7	-30.9	-34.2	-9.7	-7.9	-6.0	-5.0
Book-to-market	-10.3	-13.0	-13.6	-8.0	-10.4	-13.1	-4.2	-5.4	-6.2
Net equity expansion	-4.3	-5.0	-9.5	-5.6	-9.7	-23.0	-7.8	-11.0	-15.2
T-bill rate	-2.7	-3.5	-4.7	-2.8	-2.8	-2.0	-2.9	-2.9	-2.0
Long term yield	-7.2	-11.2	-16.1	-3.1	-3.0	-3.5	-3.2	-3.4	-4.0
Term Spread	-1.5	0.2	5.2	-3.4	-2.4	0.6	-3.4	-2.9	1.0
Long term return	-3.0	0.3	-0.7	-1.8	-1.5	-3.2	-4.8	-4.1	-4.6
Default yield spread	-9.6	-9.7	-6.6	-4.0	-4.3	-3.0	-6.6	-7.1	-4.0
Default return spread	-8.6	-2.5	-3.3	-14.8	-3.2	-3.4	-3.0	-3.4	-3.4
Inflation	-3.4	-1.9	-1.0	-2.9	-1.3	1.2	-3.3	-1.7	1.0
Inv/Capital ratio	-1.6	-0.3	0.9	-3.2	-1.2	3.4	-1.4	1.0	4.9
Panel B: Bivariate Regression with Relative Information Quality: $-1/Q_t^{-2}$									
cay	-2.6	1.0	5.3	0.9	6.9	14.1	3.4	12.2	20.6
Dividend price ratio	0.5	7.7	16.7	4.1	14.4	23.7	7.5	19.3	29.1
Dividend payout ratio	-4.8	1.1	6.6	-2.9	4.4	14.3	0.6	10.1	19.2
Dividend yield	1.2	9.8	16.6	4.2	13.7	23.4	7.2	19.1	29.7
Earning price ratio	-4.9	-2.2	3.8	-3.3	0.7	5.1	0.9	5.7	12.3
Stock volatility	-14.7	-11.1	7.2	-20.4	-16.6	8.6	0.7	10.3	14.2
Book-to-market	-3.4	1.5	8.1	-2.2	3.3	9.7	7.7	18.1	24.6
Net equity expansion	3.8	8.8	7.8	2.5	4.9	-1.1	2.0	5.0	3.9
T-bill rate	-3.1	-4.3	-1.0	1.8	7.6	13.3	4.5	12.2	16.3
Long term yield	-5.4	-7.0	-8.2	1.9	7.9	14.8	4.6	12.3	17.5
Term Spread	0.9	4.1	10.5	1.3	7.0	12.3	4.6	11.6	14.6
Long term return	2.0	9.5	11.5	2.5	8.6	12.6	3.1	10.3	14.1
Default yield spread	-1.9	5.1	11.7	1.3	8.8	17.0	2.4	11.9	20.0
Default return spread	-2.6	5.8	8.7	-8.4	6.1	9.9	4.3	10.8	13.5
Inflation	-0.9	2.3	4.8	1.2	5.5	11.1	7.0	12.3	15.2
Inv/Capital ratio	2.8	9.4	14.3	2.3	9.4	17.3	6.3	15.6	22.1

Table 10: **Cross-sectional Regression (two-stage).**

The table reports the second stage regression result of two-stage cross-sectional regression. The two-stage cross-sectional regression is equivalent to the Fama-MacBeth regression with the constant betas except for standard errors of market price of risk. Following Ozoguz (2009), the test assets are the excess returns of ten size-sorted portfolios, ten Book-to-Market-sorted portfolios, and ten momentum-sorted portfolios. Time-series of MktRf, SMB, HML, and MOM are taken from Kenneth French's website. Factors and excess returns are quarterly data from 1968:4Q to 2011:2Q. The regression coefficients are reported as $\lambda \times 100$. Below the coefficient estimates, t-statistics are reported in parentheses. The table also reports the mean absolute deviation (MAD) and the root mean squared errors (RMSE) of the pricing errors (α_i) which are the intercepts of the second stage regression. They are both multiplied by 1000. At the last column, p-value of χ^2-statistic ($\mathcal{J}$-statistic) is reported. Both t-statistics and p-value are corrected as suggested by Shanken (1992) to reflect uncertainty of the first stage estimation. The data used in the cross-sectional regression are from 1973:Q1 to 2010:Q4.

Model	Intcpt	Info.Q $\Delta \log q_t$	UNC $\Delta \log U_t$	News y_t	MktRf	SMB	HML	MOM	R^2(%) (adj.)	MAD (RMSE)	p-val (%)
(1)	0.2			1.6					24.5	3.9	50.7
	(0.4)			(1.8)					(21.8)	(5.7)	
(2)	1.6		18.2						53.2	3.6	97.1
	(16.9)		(1.0)						(51.5)	(4.5)	
(3)	2.7	6.6							17.1	4.9	0.9
	(5.2)	(1.1)							(14.1)	(6.0)	
(4)	0.8		15.6	0.7					59.3	3.4	97.6
	(2.1)		(1.5)	(0.7)					(56.3)	(4.2)	
(5)	0.8	9.2		2.2					66.7	3.1	97.4
	(2.4)	(1.1)		(2.2)					(64.2)	(3.8)	
(6)	2.3	14.3	19.1						63.4	3.1	96.7
	(9.1)	(1.4)	(1.6)						(60.7)	(4.0)	
(7)	1.1	13.6	14.6	1.5					83.3	2.2	99.0
	(4.4)	(1.3)	(1.4)	(1.5)					(81.4)	(2.7)	
(8)	2.0	10.6	12.7	1.4	-0.4	0.5	0.6		88.6	1.9	91.7
	(4.4)	(1.7)	(2.0)	(1.4)	(-0.6)	(0.9)	(1.0)		(85.7)	(2.2)	
(9)	4.7				-2.9	0.7	0.4		65.2	2.7	0.0
	(10.1)				(-4.2)	(1.4)	(0.7)		(61.2)	(3.9)	
(10)	1.6	8.0	8.3	0.8	0.1	0.5	0.6	0.3	91.4	1.6	63.4
	(3.3)	(2.1)	(1.9)	(1.4)	(0.1)	(1.0)	(1.1)	(1.1)	(88.6)	(1.9)	
(11)	2.9				-1.1	0.8	0.5	0.9	80.7	2.1	8.4
	(6.4)				(-1.5)	(1.4)	(0.9)	(2.0)	(77.6)	(2.9)	

References

Ai, H. 2010. Information quality and long-run risk: Asset pricing implications. *Journal of Finance* 65 (4):1333–1367.

Anderson, E.W., E. Ghysels, and J.L. Juergens. 2009. The impact of risk and uncertainty on expected returns. *Journal of Financial Economics* 94 (2):233–263.

Andreou, E., E. Ghysels, and A. Kourtellos. 2010. Should macroeconomic forecasters use daily financial data and how? *University of Cyprus Working Papers in Economics* .

Bansal, R. and I. Shaliastovich. 2009. Confidence risk and asset prices. Working paper.

Bansal, R. and A. Yaron. 2004. Risks for the Long Run: A Potential Resolution of Asset Pricing Puzzles. *Journal of Finance* 59 (4):1481–1509.

Bansal, R., D. Kiku, and A. Yaron. 2010. Long-Run Risks, the Macroeconomy, and Asset Prices. *American Economic Review* 100 (2):542–546.

Boguth, O. and L.A. Kuehn. 2009. Consumption volatility risk. In *EFA 2009 Bergen Meetings Paper*.

Brennan, M.J. and Y. Xia. 2005. Tay's as good as cay. *Finance Research Letters* 2 (1):1–14.

Brevik, F. and S. d'Addona. 2010. Information quality and stock returns revisited. *Journal of Financial and Quantitative Analysis* 45 (06):1419–1446.

Buraschi, A., F. Trojani, and A. Vedolin. 2009. When uncertainty blows in the orchard: Comovement and equilibrium volatility risk premia. In *European Finance Association Conference*, 19–22.

Carvalho, Carlos M., Michael Johannes, Hedibert F. Lopes, and Nicholas Polson. 2007. Particle Learning and Smoothing. University of Chicago and Columbia Univerisity.

Caskey, J.A. 2009. Information in equity markets with ambiguity-averse investors. *Review of Financial Studies* 22 (9):3595–3627.

Cochrane, J.H. 2008. The dog that did not bark: A defense of return predictability. *Review of Financial Studies* 21 (4):1533–1575.

———. 2011. Presidential address: Discount rates. *The Journal of Finance* 66 (4):1047–1108.

Collard, F., S. Mukerji, K. Sheppard, and J.M. Tallon. 2011. Ambiguity and the historical equity premium Working paper.

Croce, M.M., M. Lettau, and S. Ludvigson. 2007. Investor Information, Long-Run Risk, and the Duration of Risky Cash-Flows. Working paper.

David, A. 2008. Heterogeneous beliefs, speculation, and the equity premium. *The Journal of Finance* 63 (1):41–83.

David, A. and P. Veronesi. 2008. Inflation and Earnings Uncertainty and Volatility Forecasts: A Structural Form Approach Working paper.

Diether, K.B., C.J. Malloy, and A. Scherbina. 2002. Differences of opinion and the cross section of stock returns. *The Journal of Finance* 57 (5):2113–2141.

Dopke, J. and U. Fritsche. 2006. When do forecasters disagree? An assessment of German growth and inflation forecast dispersion. *International Journal of Forecasting* 22 (1):125–135.

Doucet, A., N. de Freitas, and N. J. Gordon. 2001. *Sequential Monte Carlo Methods in Practice*. New York: Springer-Verlag.

Drechsler, I. 2008. Uncertainty, time-varying fear, and asset prices Working paper.

Ellsberg, D. 1961. Risk, ambiguity, and the Savage axioms. *The Quarterly Journal of Economics* 643–669.

Epstein, L. G. and M. Schneider. 2008. Ambiguity, Information Quality and Asset Pricing. *Journal of Finance* LXIII (1):197–228.

Epstein, L. G. and S.E. Zin. 1989. Substitution, risk aversion and the temporal behavior of consumption and asset returns: A theoretical framework. *Econometrica* 57:937–969.

Epstein, L.G. and M. Schneider. 2003. Recursive multiple-priors. *Journal of Economic Theory* 113 (1):1–31.

———. 2007. Learning under ambiguity. *The Review of Economic Studies* 74 (4):1275.

Eraker, B., M. Johannes, and N. Polson. 2003. The Impact of Jumps and Volatility in Returns. *Journal of Finance* 58 (3):1269–1300.

Ghysels, E. and J.H. Wright. 2009. Forecasting professional forecasters. *Journal of Business and Economic Statistics* 27 (4):504–516.

Gilks, Walter R. and Carlo Berzuini. 2001. Following a Moving Target-Monte Carlo Inference for Dynamic Bayesian Models. *Journal of the Royal Statistical Society. Series B* 63 (1):127–146.

Godsill, S. J., A. Doucet, and M. West. 2004. Monte Carlo Smoothing for Nonlinear Time Series. *Journal of the American Statistical Association* 99:156–168.

Gollier, C. and E. Schlee. 2011. Information And The Equity Premium. *Journal of the European Economic Association* .

Goyal, A. and I. Welch. 2008. A comprehensive look at the empirical performance of equity premium prediction. *Review of Financial Studies* 21 (4):1455–1508.

Hansen, LP. 2007. Beliefs, Doubts and Learning: Valuing Economic Risk. *American Economic Review* 97 (2).

Hansen, L.P. and T.J. Sargent. 2007. Recursive robust estimation and control without commitment. *Journal of Economic Theory* 136 (1):1–27.

———. 2011. Robustness and ambiguity in continuous time. *Journal of Economic Theory* .

Hansen, L.P., J.C. Heaton, and N. Li. 2008. Consumption strikes back? Measuring long-run risk. *Journal of Political Economy* 116 (2):260–302.

Hess, D., N. Hautsch, and C. Müller. 2010. Price Adjustment to News with Uncertain Precision. In *EFA 2009 Bergen Meetings Paper*.

Jacoby, G., A. Paseka, and Y. Wang. 2011. The IQCAPM: Asset Pricing with Information-Quality Risk .

Johannes, M., A. Korteweg, and N. Polson. 2009a. Sequential learning, predictive regressions, and optimal portfolio returns. In *AFA 2009 San Francisco Meetings Paper*.

Johannes, M., N. Polson, and S.M. Yae. 2009b. Quantile filtering and learning. Working paper.

Johannes, M., L. Lochstoer, and Y. Mou. 2010. Learning about Consumption Dynamics Working paper.

Johannes, Michael and Nicholas Polson. 2007. Particle Filtering and Learning. Working paper.

Johannes, M.S., N. Polson, and S.M. Yae. 2009c. Sequential Inference for Nonlinear Models Working paper.

Johnson, T.C. 2004. Forecast dispersion and the cross section of expected returns. *The Journal of Finance* 59 (5):1957–1978.

Jostova, G. and A. Philipov. 2005. Bayesian analysis of stochastic betas. *Journal of Financial and Quantitative Analysis* 40 (4):747.

Ju, N. and J. Miao. 2012. Ambiguity, learning, and asset returns. *Econometrica* , forthcoming.

Kelly, Bryan T. 2011. Tail Risk and Asset Prices. Working paper.

Kelly, Bryan T. and S. Pruitt. 2011a. Market Expectations in the Cross Section of Present Values. Working paper.

———. 2011b. The Three-Pass Regression Filter: A New Approach to Forecasting with Many Predictors. Working paper.

Klibanoff, P., M. Marinacci, and S. Mukerji. 2005. A smooth model of decision making under ambiguity. *Econometrica* 73 (6):1849–1892.

———. 2009. Recursive smooth ambiguity preferences. *Journal of Economic Theory* 144 (3):930–976.

Lahiri, K. and X. Sheng. 2010. Measuring forecast uncertainty by disagreement: The missing link. *Journal of Applied Econometrics* 25 (4):514–538.

Lamont, O.A. 2002. Macroeconomic forecasts and microeconomic forecasters. *Journal of economic behavior & organization* 48 (3):265–280.

Leippold, M., F. Trojani, and P. Vanini. 2008. Learning and asset prices under ambiguous information. *Review of Financial Studies* 21 (6):2565–2597.

Li, G. 2005. Information quality, learning, and stock market returns. *Journal of Financial and Quantitative Analysis* 40 (03):595–620.

Liu, J. and M. West. 2001. *Combined Parameter and State Estimation in Simulation-based Filtering*, chap. 10. Sequential Monte Carlo in Practice. New York: Springer-Verlag. 197-223.

Liu, Jun S. 2001. *Monte Carlo Strategies in Scientific Computing*. New York: Springer-Verlag.

Opp, C.C.G. 2008. Intertemporal Information Acquisition and Asset Market Dynamics .

Ozoguz, A. 2009. Good Times or Bad Times? Investors' Uncertainty and Stock Returns. *Review of Financial Studies* 22 (11):4377–4422.

Pakos, M. and H. Chen. 2008. Asset Pricing with Uncertainty for the Long Run Working paper.

Pastor, L. and R.F. Stambaugh. 2009. Predictive systems: Living with imperfect predictors. *The Journal of Finance* 64 (4):1583–1628.

Patton, A.J. and A. Timmermann. 2008. Predictability of output growth and inflation: A multi-horizon survey approach. *Journal of Business and Economic Statistics* 1–14.

———. 2010. Why do forecasters disagree? Lessons from the term structure of cross-sectional dispersion. *Journal of Monetary Economics* .

Pitt, M. K. and N. Shephard. 1999. Filtering via Simulation: Auxiliary Particle Filter. *Journal of the American Statistical Association* 94:590–599.

Scheinkman, J.A. and W. Xiong. 2003. Overconfidence and speculative bubbles. *Journal of Political Economy* 111 (6):1183–1220.

Shaliastovich, I. 2008. Learning, confidence and option prices. Tech. rep., working paper, Duke University. Working paper.

Storvik, G. 2002. Particle filters in state space models with the presence of unknown static parameters. *IEEE Transactions on Signal Processing* 50:281–289.

Subramanyam, KR. 1996. Uncertain precision and price reactions to information. *Accounting Review* 207–219.

Ulrich, M. 2010. Observable long-run ambiguity and long-run risk Working paper.

Van Binsbergen, J.H. and R.S.J. Koijen. 2010. Predictive Regressions: A Present-Value Approach. *The Journal of Finance* 65 (4):1439–1471.

Vanden, J.M. 2008. Information quality and options. *Review of Financial Studies* 21 (6):2635.

Veronesi, P. 2000. How does information quality affect stock returns? *Journal of Finance* 807–837.

Yae, Seung M. 2011a. Extracting, Forecasting, and Testing of Learning-related Factors. Working paper.

———. 2011b. Foresight, Hindsight, and Learning with Ambiguity. Working paper.